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Analysis of Absolute Stability for Time-delay Teleoperation Systems

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Abstract: In this paper, a new bilateral control algorithm based on absolute stability theory is put forward, which aims at the time-delay teleoperation system with force feedback from the slave directly. In the new control algorithm, the delay-dependent stability, instead of delay-independent stability, is taken as the aim of control design. It improves the transparency of the system at the price of unnecessary stability. With this algorithm, the time-delay teleoperation systems have good transparency and stability. A simulation system is established to verify the effect of this algorithm.

Keywords: Time-delay, stability, transparency, absolutely stability, delay-dependent stability.

1 Introduction

Teleoperation is composed of master and slave robot mechanisms, as shown in Fig. 1. The operator controls the slave and feels the force between the slave and environment through the master. The slave follows the movement of the master and feedbacks the force that the slave acts on environment. It enables the operator to manipulate and interact with remote environment. Its applications range from space and underwater exploration, the handling of radioactive materials, to the microsurgery, *etc.* When the master and slave are far away, these teleoperation systems often face a key challenge: the time delay between the master and slave.

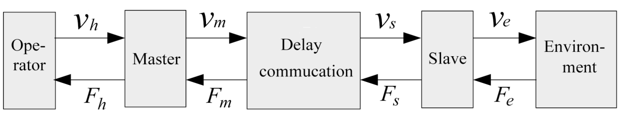

Fig. 1 Sketch of bilateral control with fore-feedback

Force feedback provides operators with more complete information and increases their sense of being present at the slave site. When the force feedback exists, the teleoperation system is a two port network and called bilateral system. However, the force feedback renders the close-loop system very sensitive to delay. Even small amounts of delay can drive the force feedback system unstable. The delay existing in network teleoperation system makes the force feedback impossible without compensation.

As in any other control system, stability is the most important issue in bilateral teleoperation systems with delay and has been studied with many theories. Most previous stability studies were based on passivity theory, such as scattering operators[1], wave variables[2−4]. Different algorithms such as slide-mode[5], absolute stability[6−8], *etc.* also have been used in stability analysis and controller design.

Operating performance is another important issue in delayed bilateral systems. In this paper, we use transparency to measure the performance of system. The more transparent the system is, the better performance the system has. However, Lawrence[9] has pointed out that the stability and transparency are conflict each other, the improvement of one will worsen the other one. The previous algorithms such as scattering operators and wave variables didn't consider the transparency. Its stability is robust to the delay and called delay-independent stability. However, the transparency of these systems is poor. In many occasion, the delay is upper-limited, it is unnecessary to require the stability robust to any delay. If we take delay-dependent stability as the stability criterion, instead of delay-independent stability, the transparency should be better. Papers [6-8] also used the absolutely stability theory as the control tool. However, it feedbacked slave position from slave to master, but didn't feedback the force that the slave acted on environment directly. In this manner, the delay from slave to master makes the transparency worse, for the force that the operator feels is PI of the error between $V_m(t)$ and $V_s(t - T_2)$, which isn't the force between the slave and environment. If the force between the slave and environment is feedbacked directly, the transparency should be better.

This paper analyses the delayed bilateral system with absolute stability theory. The system feedbacks the force between the slave and environment directly, instead of the slave's position. A delay-dependent stability is achieved. This algorithm can improve the transparency and maintain the stability simultaneously. The absolute stability of two-port network system is discussed in Section 2 using Llewellyn's criteria. In Section 3, the stability and transparency of the system are analysed based on absolute stability theory and resistance theory separately. Section 4 shows the simulation result, followed by conclusion in Section 5.

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2 Absolute stability theory

When a two-port network is stable with all possible passive terminations, it is said to be absolutely stable. A passivity system is absolutely stable. But a absolutely stable system may be active. A condition for absolute stability is Llewellyn's criterion[10],suggested by Hashtrudizaad and Salcudean for stability analysis in a teleoperation system[11].

Theorem 1. (Llewellyn's criterion) A two-port network $Z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$ is absolutely stable if and only if

1) $z_{11}(s)$ and $z_{22}(s)$ have no poles in the right half plane;

2) Any poles of $z_{11}(s)$ and $z_{22}(s)$ on the imaginary axis are simple, with real and positive residues;

3) For all real values of ω we have

$$\mathrm{Re}(z_{11}) > 0 \tag{1}$$

$$\mathrm{Re}(z_{22}) > 0 \tag{2}$$

$$\frac{2\mathrm{Re}(z_{11})\,\mathrm{Re}(z_{22}) - \mathrm{Re}(z_{12}z_{21})}{|z_{12}z_{21}|} > 1 \tag{3}$$

where $|\cdot|$ and $\mathrm{Re}(\cdot)$ denote the absolute and real values of their corresponding arguments.

3 Theory analysis

3.1 System model

The bilateral teleoperation system model is shown as follows.

Dynamic model of the master:

$$M_m \dot{V}_m(t) + B_m V_m(t) = F_h(t) - F_s(t - T_2). \tag{4}$$

Dynamic model of the slave:

$$M_s \dot{V}_s(t) + B_s V_s(t) = F_s(t) - F_e(t) \tag{5}$$

F_s is virtual force, the model is

$$F_s = K_c (X_m(t - T_1) - X_s(t)) \tag{6}$$

X_m,X_s are the positions of master and slave, separately;
M_m,M_s are the qualities of master and slave, separately;
B_m,B_s are the damps of master and slave, separately;
F_h is the force that the operator acts on the master;
F_e is the force between the slave and environment;
T_1 is the delay from master to slave;
T_2 is the delay from slave to master;
K_c is the proportion parameter of slave position-error-based PID controller.

From(4)$\sim$(6), the system model is given by (7).

$$\begin{bmatrix} F_h \\ F_e \end{bmatrix} = \begin{bmatrix} M_m s + B_m + \frac{K_c}{s} e^{-s(T_1+T_2)} & \frac{K_c}{s} e^{-sT_2} \\ \frac{K_c}{s} e^{-sT_1} & M_s s + B_s + \frac{K_c}{s} \end{bmatrix} \begin{bmatrix} V_m \\ -V_s \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} V_m \\ -V_s \end{bmatrix} \tag{7}$$

3.2 Stability analysis

From (7), we have

$$Z_{11}(s) = M_m s + B_m + \frac{K_c}{s} e^{-s(T_1+T_2)} \tag{8}$$

$$Z_{22}(s) = M_s s + B_s + \frac{K_c}{s} \tag{9}$$

Obviously, both z_{11} and z_{22} have only poles at $s = 0$,which are on the imaginary axis and simple, with real and positive residue K_c , thus the first and second conditions are satisfied. Now, we consult the third condition.

From (7), we have

$$\mathrm{Re}(z_{11}) = \mathrm{Re}\left(M_m s + B_m + \frac{K_c}{s} e^{-s(T_1+T_2)}\right) = B_m - \frac{K_c}{\omega} \sin\omega(T_1 + T_2) \tag{10}$$

$$\mathrm{Re}(z_{22}) = \mathrm{Re}\left(M_s s + B_s + \frac{K_c}{s}\right) = \mathrm{Re}\left(M_s j\omega + B_s + \frac{K_c}{j\omega}\right) = B_s \tag{11}$$

$$\frac{2\mathrm{Re}(z_{11})\,\mathrm{Re}(z_{22}) - \mathrm{Re}(z_{12}z_{21})}{|z_{12}z_{21}|} = \frac{2\left(B_m - \frac{K_c}{\omega}\sin\omega(T_1+T_2)\right)B_s}{\left|\left(\frac{K_c}{j\omega}\right)^2 e^{-s(T_1+T_2)}\right|} - \frac{\mathrm{Re}\left(\left(\frac{K_c}{j\omega}\right)^2 e^{-s(T_1+T_2)}\right)}{\left|\left(\frac{K_c}{j\omega}\right)^2 e^{-s(T_1+T_2)}\right|} \geq 1. \tag{12}$$

Compare (10) with (1), the condition of Llewellyn's criterion 3.1 is

$$B_m > \frac{K_c}{\omega} \sin\omega(T_1 + T_2). \tag{13}$$

Compare (11) with (2), the condition of Llewellyn's criterion 3.2 is

$$B_s > 0. \tag{14}$$

Compare (12) with (3), the condition of Llewellyn's criterion 3.3 is

$$B_mB_s \geq {K_c}^2\frac{(T_1+T_2)^2}{4} + \frac{K_cB_s}{2\omega}\sin\omega\,(T_1+T_2)\,. \tag{15}$$

Condition (14) is simply. The control parameter can get from it directly. However, conditions (13) and (15) are complex. They depend on the frequency ω. The control parameter can't get from them directly. We have to check conditions (13) and (15) for all frequencies.

Firstly, we discuss condition (13). Suppose $y_1 = \frac{K_c}{\omega}\sin\omega\,(T_1+T_2)$, $K_c=1$, $T_1=1S$, $T_2=1S$. Fig. 2 shows the left and right sides of this inequality with the controller parameters given above. From Fig. 2, we can find that the right side of the inequality converges to a constant. So we can find a fit B_m ,which makes (13) established for any delay and frequency.

Then, we discuss condition (15). Suppose $y_2 = {K_c}^2\frac{(T_1+T_2)^2}{4} + \frac{K_cB_s}{2\omega}\sin\omega\,(T_1+T_2)$, $K_c=1$, $T_1=1S$, $T_2=1S$, Fig. 3 show the left and right sides of this inequality with the controller parameters given above. From Fig. 3, we can find that the right side of the inequality converges to a constant. So we can find a fit K_c which makes (15) established for any delay and frequency.

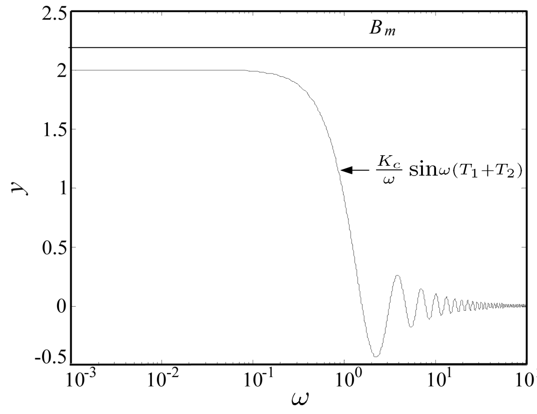

Fig. 2 The curve of y_1 about ω

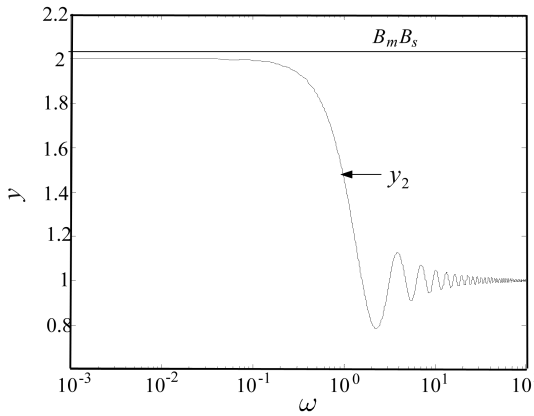

Fig. 3 The curve of y_2 about ω

In Summary, we can find suitable parameters which satisfy the three conditions of Llewellyn's criterion, and guarantee the stability of the system. Checking conditions (13) and (15), we find that the control parameters depend on the delay. A given parameter only can guarantee the stability of the system whose delay is less than a given value. This stability is called delay-dependent stability. If the control parameter of K_c is small enough, the system is always stable. However, when K_c is too small, the transparency of system will get worse. So we must consider the stability and transparency at the same time.

3.3 Transparency analysis

In analogy to an electrical network, we use impedance to measure the transparency. When the operator's impedance $Z_h = F_h/V_m$ is equal to the environment impedance $Z_e = F_e/V_s$, the bilateral control system is transparent completely.

From (7), we have

$$Z_h = Z_{11} - \frac{Z_{12}Z_{21}}{Z_e+Z_{22}}. \tag{16}$$

Suppose $Z_m = M_ms+B_m$, $Z_s = M_ss+B_s$, $Z_f = K_c/s$ are the damps of master, slave, PID controller of slave, respectively.

From (16), we have

$$Z_h = Z_m + Z_fe^{-s(T_1+T_2)} - \frac{{Z_f}^2}{Z_e+Z_s+Z_f}e^{-s(T_1+T_2)} = Z_m + \frac{(Z_e+Z_s)\,Z_fe^{-s(T_1+T_2)}}{Z_e+Z_s+Z_f}. \tag{17}$$

Then, we analyse the system transparency on two extremely situations:

1) The slave moves freely, $Z_e \to 0$.

To have good transparency, $Z_h \to 0$ must be satisfied, for $Z_e \to 0$.

From (17), we have

$$Z_h = Z_m + \frac{Z_sZ_fe^{-s(T_1+T_2)}}{Z_s+Z_f}. \tag{18}$$

Because $Z_s > 0$, $Z_f > 0$ and $Z_h > Z_m$ is always true, the system couldn't be transparent completely. To have good transparency,

$$\frac{Z_sZ_fe^{-s(T_1+T_2)}}{Z_s+Z_f} \to 0. \tag{19}$$

For a given environment, Z_s is constant. So, the smaller Z_f, the better transparent the system.

2) Hard contact between the slave and environment, $Z_e \to \infty$.

To have good transparency, $Z_h \to \infty$ must be satisfied, for $Z_e \to \infty$.

From (17), we have

$$Z_h = Z_m + \frac{\left(1+\frac{z_s}{z_e}\right)Z_fe^{s(T_1+T_2)}}{1+\frac{Z_s}{Z_e}+\frac{Z_f}{Z_e}} \approx Z_m + Z_fe^{s(T_1+T_2)}$$

For a given environment, Z_m is constant. So, the bigger Z_f, the better transparency the system has.

In summary, for a bilateral control system with force feedback directly, control parameters must satisfy the following rules:

1) Under any condition, all parameters must satisfy (14), Figs. 2 and 3. This rule guarantees the stability of the system.

2) When the slave moves freely, the proportion gain of slave position-error-based PID controller should be as small as possible. The force feedback to the operator is small.

3) When the slave is hard contact with environment, the proportion gain of slave position-error-based PID controller should be as big as possible. The force feedback to the operator is big.

Rule 1 ensures the stability of system. Rule 2 and rule 3 guarantee the transparency of system. A bilateral control system satisfying above rules can make the system have good stability and transparency simultaneity.

4 Simulation result

In order to illustrate and verify the conclusions of the previous sections, we established a simulation system with the SinMechanics of MATLAB[12,13]. Master is force control, slave is position control. Velocity command which is from master to slave and force command which is from slave to master are transported directly. In each simulation, the master rigidly follows a preset trajectory. The slave moves in free space, then interacts with a rigid wall, and finally returns into free space.

The basic parameters used in this simulation system are shown in Table 1. The value of proportion gain of slave position-error-based PID controller isn't constant. When the slave moves freely, the proportion gain of slave position-error-based PID controller is Kc. When the slave is hard contact with the environment, the proportion gain of slave position-error-based PID controller is $15Kc$. When Kc are 0.03, 0.5 or 3, the corresponding experiment results are shown in Figs. 4∼6. In each figure, the top one is the positions of the master and slave. The shallow is master's, and the deep is slave's. The lower one is the force that the operator feels.

Table 1 Basic control parameters

Parameters	Valus
T_1	1 s
T_2	1 s
$Max\,\|x_s\|$	0.4 m
M_m	3.894 2 kg
M_s	3.894 2 kg
B_m	0.1 kg/s
B_s	0.1 kg/s

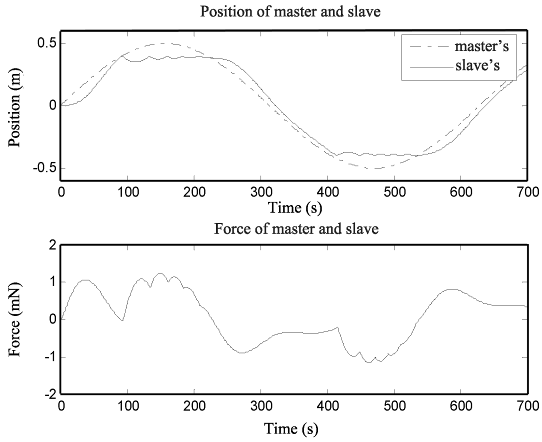

Fig. 4 Force and position profiles of master and slave(K_c=0.03)

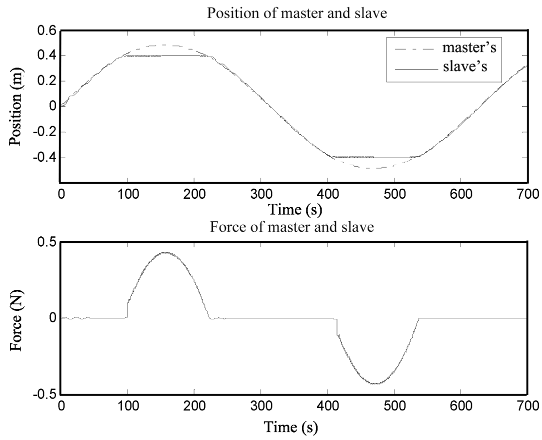

Fig. 5 Force and position profiles of master and slave(K_c=0.5)

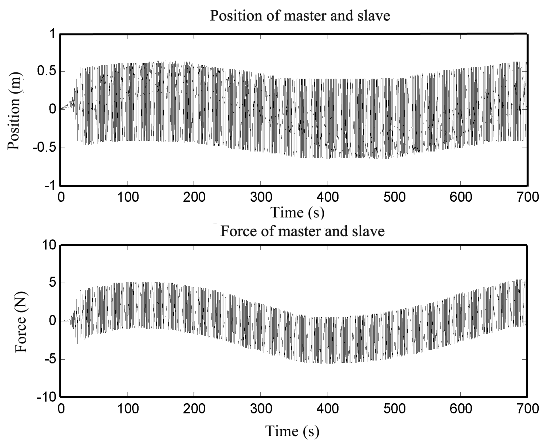

Fig. 6 Force and position profiles of master and slave(K_c=3)

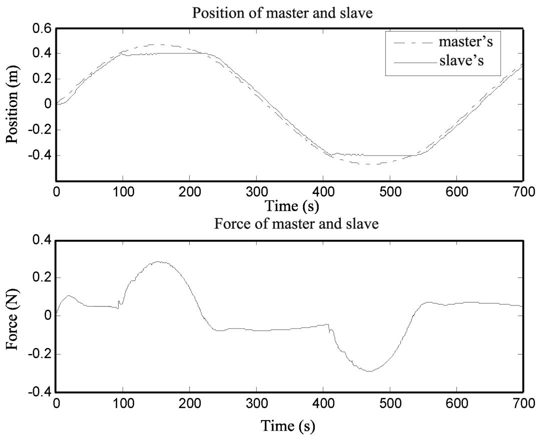

Fig. 7 Force and position profiles of master and slave.(feedback position from slave to master)

Figs. 4∼6 show that when the slave control parameter is too big, system is unstable. When the slave control parameter is small enough, system is always stable. However, when the slave control parameter is too small, the transparency of system is worse. Comparing Fig. 5 and Fig. 7, we can find that the transparency of the system with force feedback directly is better than that of the system with position feedback. When the slave moves freely, in the system with force feedback directly, the force operator feels is almost zero; in the system with position feedback, the force operator feels can't be ignored.

5 Conclusion

In this paper, we put forward a new bilateral control algorithm. It feedbacks the force between the slave and environment directly, instead of the position of slave. This algorithm is based on absolute stability. The stability of system is delay-dependent, which is less robust than delay-independent stability. However, the delay generally is upper-limited, the stability for any delay is unnecessary. This algorithm improves the transparency of system at the price of unnecessary stability. Comparing to the system with position feedback, this algorithm can improve transparency greatly. Simulation experiment has verified the effect of this algorithm.

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