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A Model-free Approach to Fault Detection of Continuous-time Systems Based on Time Domain Data

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Abstract: In this paper, a model-free approach is presented to design an observer-based fault detection system of linear continuous-time systems based on input and output data in the time domain. The core of the approach is to directly identify parameters of the observer-based residual generator based on a numerically reliable data equation obtained by filtering and sampling the input and output signals.

Keywords: Fault detection, linear continuous time-invariant systems, time domain data, subspace methods, observer-based residual generator.

1 Introduction

With the increasing requirement of modern complex control systems on safety and reliability, fault detection (FD) technique has received much attention since 1970′s and achieved considerable theoretical development[1−5]. Applications have been found in process industry, aerospace and aeronautics, automobile industry, transportation systems, *etc.* The basic idea of model-based FD is to generate analytical redundancy with the help of mathematical model of supervised systems. Observer-based FD is one of the most important kinds of model-based FD approaches[2, 3, 5]. The central part of an observer-based FD system is an output observer. The fault-indicating signal, usually called residual, is obtained by comparing the measured outputs with their estimations. Often a model of the supervised system is assumed to be available for the design of the output observer. If it is not the case, then an identification of the system model is usually carried out before an observer-based FD system is designed. On the other side, data-driven monitoring methods such as principle component analysis (PCA) and partial least square (PLS) have gained acceptance in the process industry (see [6–8] and the references therein).

Recently, based on the subspace identification technique[9, 10], a kind of new data-driven approaches have been developed by [11, 12], which can obtain a model-based residual generator directly from system input and output signals without identifying the model of the system at first. Compared with the conventional two-step approach, this new line of design approaches can save design efforts. Though it seems straightforward that the basic idea of this kind of subspace based model-free design approach can be extended to continuous-time systems by differentiating the time domain data, such a procedure is sensitive to noise and may be numerically unstable. To cope with this problem, an approach was suggested in [13], whose core is to use the frequency domain data instead of the time domain data.

In this paper, we will propose another approach to construct an observer-based FD system using only the time domain input and output data collected on the system. The difficulty of differentiation is overcome by filtering the input and output data before applying subspace technique. To this aim, an important relation between key matrices of the filtered system and the original system is derived. Based on it, the parity space of the original system can be retrieved and after that an observer-based residual generator is obtained. The proposed algorithm consists of three parts: 1) identification of the key matrices related to the parity space of the filtered system, 2) recover the key matrices of the original system, 3) design of an observer-based residual generator.

2 Problem formulation

Consider a continuous linear time-invariant (LTI) system described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + E_f f(t) \\ y(t) &= Cx(t) + Du(t) + F_f f(t) \end{aligned} \tag{1}$$

where $x \in \mathbf{R}^n$ denotes the state vector, $u \in \mathbf{R}^{k_u}$ the control input vector, $y \in \mathbf{R}^m$ the measured output vector, $f \in \mathbf{R}^{k_f}$ the fault vector, $A \in \mathbf{R}^{n\times n}, B \in \mathbf{R}^{n\times k_u}, C \in \mathbf{R}^{m\times n}, D \in \mathbf{R}^{m\times k_u}, E_f \in \mathbf{R}^{n\times k_f}, F_f \in \mathbf{R}^{m\times k_f}$ are constant matrices of compatible dimensions.

It is known that the first step of fault detection is residual generation. For this purpose, an observer based residual generator can be designed as[4]

$$\begin{aligned} \dot{z}(t) &= Gz(t) + Ju(t) + Ly(t) \\ r(t) &= wz(t) + pu(t) + vy(t) \end{aligned} \tag{2}$$

where G is stable and further the following equations

$$\begin{aligned} TA - GT &= LC, \quad vC + wT = 0 \\ TB - LD &= J, \quad\ \ p + vD = 0 \end{aligned} \tag{3}$$

are satisfied. In this case, the residual dynamics is governed

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by

$$\begin{aligned} \dot{e}(t) &= Ge(t) + (LF_f - TE_f)f(t) \\ r(t) &= we(t) + vF_f f(t) \end{aligned} \quad (4)$$

where $e(t) = z(t) - Tx(t)$. The residual signal is not influenced by the control input and will deviate from zero as long as the fault f is nonzero. As a result, the fault can be detected based on the change of the residual signal.

If matrices A, B, C, and D of system (1) are known, then there are several algorithms available to solve the set of (3). In the following, a novel algorithm proposed by Ding *et al.* (1998)[14] will be briefly recapitulated in Lemma 1. At first, two key matrices H_o (depending on A, C) and H_s (depending on A, B, C, and D) are defined, respectively, as

$$H_o = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^s \end{bmatrix} \quad (5)$$

$$H_s = \begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ CA^{s-1}B & \cdots & CB & D \end{bmatrix}$$

where s is an integer, $H_o \in \mathbf{R}^{m(s+1)\times n}$ and $H_s \in \mathbf{R}^{m(s+1)\times k_u(s+1)}$.

Lemma 1. Given a vector v_s that satisfies

$$v_s H_o = 0 \quad (6)$$

partition the vectors v_s and $\rho_s = v_s H_s$ as follows

$$v_s = \begin{bmatrix} v_{s,0} & v_{s,1} & \cdots & v_{s,s} \end{bmatrix}, \quad v_{s,i} \in \mathbf{R}^{1\times m} \quad (7)$$

$$\rho_s = \begin{bmatrix} \rho_{s,0} & \rho_{s,1} & \cdots & \rho_{s,s} \end{bmatrix}, \quad \rho_{s,i} \in \mathbf{R}^{1\times k_u}. \quad (8)$$

Then the equations in (3) are satisfied by the transformation matrix

$$T = \begin{bmatrix} v_{s,1} & \cdots & v_{s,s-1} & v_{s,s} \\ v_{s,2} & \cdots & v_{s,s} & 0 \\ \vdots & & & \vdots \\ v_{s,s} & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{s-1} \end{bmatrix}$$

and the coefficient matrices

$$G = \begin{bmatrix} 0 & \cdots & 0 & g_1 \\ 1 & \cdots & 0 & g_2 \\ \vdots & & & \vdots \\ 0 & \cdots & 1 & g_s \end{bmatrix} \quad (9)$$

$$J = \begin{bmatrix} \rho_{s,0} \\ \rho_{s,1} \\ \vdots \\ \rho_{s,s-1} \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_s \end{bmatrix} \rho_s$$

$$L = -\begin{bmatrix} v_{s,0} \\ v_{s,1} \\ \vdots \\ v_{s,s-1} \end{bmatrix} - \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_s \end{bmatrix} v_{s,s}$$

$$w = \begin{bmatrix} 0 & \cdots & 0 & -1 \end{bmatrix}$$

$$p = -\rho_{s,s}$$

$$v = v_{s,s}$$

where $g_1, g_2, \cdots$, and g_s are free-selectable constants guaranteeing the stability of matrix G.

The advantage of the above algorithm is that matrices G, J, L, w, p, v satisfying (3) can be entirely determined based on the vectors v_s, ρ_s and the free parameters $g_1, g_2, \cdots, g_s$. That means for the design of an observer-based residual generator (2), the information of v_s and ρ_s is sufficient. If matrices A, B, C, D are known, then v_s, ρ_s can be computed by simply solving equation (6). If matrices A, B, C, D are unknown, it is enough to identify v_s, ρ_s instead of identifying A, B, C, D. Motivated by the lemma, an approach was introduced in [13] to identify v_s, ρ_s for continuous-time system (1) using the frequency domain data.

The problem to be solved in this paper is formulated as: Given a set of time domain input and output data of unknown system (1) in normal operations, determine an observer-based FD system.

3 Design approach

The basic idea of the paper is motivated by [15,16]. The time domain input and output data are filtered with a bank of low pass filters before the identification algorithm is applied, which is easily realizable in practice. Then the relation between the filtered system and the original system will be derived, based on which an observer-based residual generator is obtained afterwards.

3.1 Equation of filtered data

Doing a Laplacian transform of system model (1) with $f = 0$ gives

$$sX(s) = AX(s) + BU(s) \quad (10)$$

$$Y(s) = CX(s) + DU(s). \quad (11)$$

Define

$$\omega = \frac{1}{1+\tau s} \quad (12)$$

where τ is a positive number to be chosen appropriately.

Multiplying both sides of (10) by $\tau\omega$ and adding on both sides a term $X(s)$ results in

$$\begin{aligned} X(s) + \tau\omega s X(s) = X(s) + \tau\omega A X(s) + \tau\omega B U(s) = \\ (I + \tau\omega A)X(s) + \tau\omega B U(s) \\ \Rightarrow X(s) = (I + \tau\omega A - \tau\omega s I)X(s) + \tau\omega B U(s) \end{aligned}$$

As $\omega = 1 - \tau s \omega$, we have

$$X = (I + \tau A)\omega X(s) + \tau B \omega U(s). \quad (13)$$

Multiplying both sides of output equation (11) by ω^s yields

$$\omega^s Y(s) = C(\omega^s X(s)) + D(\omega^s U(s)). \tag{14}$$

Substituting (13) into

$$\omega^{s-1} Y(s) = C(\omega^{s-1} X(s)) + D(\omega^{s-1} U(s)) \tag{15}$$

allows us to get

$$\begin{aligned}\omega^{s-1} Y(s) \;\; &= \;\; C(I+\tau A)\omega^s X(s) + \\ &\quad\;\; C\tau B \omega^s U(s) + D\omega^{s-1} U(s).\end{aligned} \tag{16}$$

In this way, it can also be obtained that

$$\begin{aligned}Y(s) \;\; &= \;\; C(I+\tau A)^s \omega^s X(s) + \\ &\quad\;\; C(I+\tau A)^{s-1}\tau B \omega^s U(s) + \cdots + \\ &\quad\;\; C\tau B \omega U(s) + DU(s).\end{aligned} \tag{17}$$

Summarizing (14∼17) gives the following input-output equation

$$\begin{bmatrix} \omega^s Y(s) \\ \omega^{s-1} Y(s) \\ \vdots \\ Y(s) \end{bmatrix} = \begin{bmatrix} C \\ C(I+\tau A) \\ \vdots \\ C(I+\tau A)^s \end{bmatrix} \omega^s X(s) + \begin{bmatrix} D & O & \cdots & O \\ \tau CB & D & \ddots & \vdots \\ \vdots & \ddots & \ddots & O \\ \tau C(I+\tau A)^{s-1} B & \cdots & \tau CB & D \end{bmatrix} \begin{bmatrix} \omega^s U(s) \\ \omega^{s-1} U(s) \\ \vdots \\ U(s) \end{bmatrix}. \tag{18}$$

Define

$$H_{o\tau} = \begin{bmatrix} C \\ C(\tau A + I) \\ \vdots \\ C(\tau A + I)^s \end{bmatrix}$$

$$H_{s\tau} = \begin{bmatrix} D & O & \cdots & O \\ \tau CB & D & \ddots & \vdots \\ \vdots & \ddots & \ddots & O \\ \tau C(\tau A + I)^{s-1} B & \cdots & \tau CB & D \end{bmatrix}. \tag{19}$$

In the time domain, equation (18) is equivalent to

$$\begin{bmatrix} \mathcal{L}^{-1}(\omega^s Y(s)) \\ \mathcal{L}^{-1}\left(\omega^{s-1} Y(s)\right) \\ \vdots \\ \mathcal{L}^{-1}(Y(s)) \end{bmatrix} = H_{o\tau}\mathcal{L}^{-1}(\omega^s X(s)) + H_{s\tau} \begin{bmatrix} \mathcal{L}^{-1}(\omega^s U(s)) \\ \mathcal{L}^{-1}\left(\omega^{s-1} U(s)\right) \\ \vdots \\ \mathcal{L}^{-1}(U(s)) \end{bmatrix} \tag{20}$$

where $\mathcal{L}^{-1}\left(\omega^i U(s)\right)$ and $\mathcal{L}^{-1}\left(\omega^i Y(s)\right)$ denote the inverse Laplacian transforms of $\omega^j U(s)$ and $\omega^j Y(s)$, respectively.

The vectors $\mathcal{L}^{-1}\left(\omega^i U(s)\right)$, $\mathcal{L}^{-1}\left(\omega^i Y(s)\right)$, $i = 1, \cdots, s$, can be obtained by filtering the input and output signals using a low-pass filter ω^i, respectively, as shown in Fig. 1. The time constant of the filter is an important design parameter. If the value of τ is chosen to be too large, then the difference between $\mathcal{L}^{-1}\left(\omega^i U(s)\right)$ and $\mathcal{L}^{-1}\left(\omega^{i+1} U(s)\right)$ will be very small for higher order i and thus undesirable.

Based on data equation (20), matrices $H_{o\tau}^{\perp}$ and $H_{o\tau}^{\perp} H_{s\tau}$ can be identified with the technique similar to [12,13]. In the subsequent subsection, we will show how to identify matrices $H_{o\tau}^{\perp}$, $H_{o\tau}^{\perp} H_{s\tau}$ and after that how to recover matrices $H_o^{\perp}$, $H_o^{\perp} H_s$ from $H_{o\tau}^{\perp}$, $H_{o\tau}^{\perp} H_{s\tau}$.

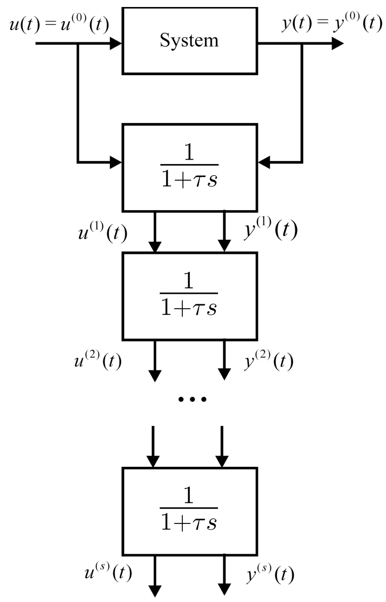

Fig. 1 Procedure of obtaining data matrices

3.2 Identification of $H_{o\tau}^{\perp}$ and $H_{o\tau}^{\perp} H_{s\tau}$

Assume that N samples of the filtered signals $\mathcal{L}^{-1}\left(\omega^i U(s)\right)$, $\mathcal{L}^{-1}\left(\omega^i Y(s)\right)$ at time $t_1, \cdots, t_N$ are given. For convenience, we denote

$$\begin{aligned} y^{(i)}(t) \;\; &= \;\; \mathcal{L}^{-1}\left(\omega^i Y(s)\right), \; y^{(i)}(k) = y^{(i)}(t)\,|_{t=t_k} \\ u^{(i)}(t) \;\; &= \;\; \mathcal{L}^{-1}\left(\omega^i U(s)\right), \; u^{(i)}(k) = u^{(i)}(t)\,|_{t=t_k} \\ x^{(s)}(t) \;\; &= \;\; \mathcal{L}^{-1}(\omega^s X(s)), \; x^{(s)}(k) = x^{(s)}(t)\,|_{t=t_k} \end{aligned}$$

for $i = 0, 1, \cdots, s$ and $k = 1, 2, \cdots, N$. Considering (20) at each sample, we get

$$Y_N = H_{o\tau} X_N + H_{s\tau} U_N \tag{21}$$

with $Y_N \in \mathbf{R}^{(s+1)m\times N}$, $U_N \in \mathbf{R}^{(s+1)k_u\times N}$, $X_N \in \mathbf{R}^{n\times N}$ defined as

$$Y_N = \begin{bmatrix} y^{(s)}(1) & y^{(s)}(2) & \cdots & y^{(s)}(N) \\ y^{(s-1)}(1) & y^{(s-1)}(2) & & y^{(s-1)}(N) \\ \vdots & \vdots & & \vdots \\ y^{(0)}(1) & y^{(0)}(2) & \cdots & y^{(0)}(N) \end{bmatrix}$$

$$X_N = \begin{bmatrix} x^{(s)}(1) & y^{(s)}(2) & \cdots & y^{(s)}(N) \end{bmatrix}$$

$$U_N = \begin{bmatrix} u^{(s)}(1) & u^{(s)}(2) & \cdots & u^{(s)}(N) \\ u^{(s-1)}(1) & u^{(s-1)}(2) & & u^{(s-1)}(N) \\ \vdots & \vdots & & \vdots \\ u^{(0)}(1) & u^{(0)}(2) & \cdots & u^{(0)}(N) \end{bmatrix}. \quad (22)$$

Equation (21) can be re-written as

$$W_N = HQ_N$$

where $W_N \in \mathbf{R}^{(s+1)(m+k_u)\times N}$,$Q_N \in \mathbf{R}^{(n+(s+1)k_u)\times N}$, $H \in \mathbf{R}^{(s+1)(m+k_u)\times(n+(s+1)k_u)}$, and

$$\begin{aligned} W_N &= \begin{bmatrix} Y_N \\ U_N \end{bmatrix}, \ Q_N = \begin{bmatrix} X_N \\ U_N \end{bmatrix} \\ H &= \begin{bmatrix} H_{o\tau} & H_{s\tau} \\ O & I \end{bmatrix}. \end{aligned} \quad (23)$$

Under persistently exciting input and for sufficiently large N, matrix Q_N is of full row rank. It means that the data matrix W_N and matrix H representing the structure of the system have the same left null space. According to this observation, the information of matrix H can be extracted from the data matrix W_N.

Doing the singular value decomposition (SVD) of the data matrix W_N yields

$$W_N = U\Sigma_S V \quad (24)$$

where

$$\begin{aligned} U &= \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix}, \ V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix} \\ \Sigma_S &= \begin{bmatrix} S & O \\ O & O \end{bmatrix} \\ U &\in \mathbf{R}^{(s+1)(m+k_u)\times(s+1)(m+k_u)}, \\ V &\in \mathbf{R}^{N\times N}, \ \Sigma_S \in \mathbf{R}^{(s+1)(m+k_u)\times N} \\ U_{11} &\in \mathbf{R}^{m(s+1)\times(k_u(s+1)+n)} \\ U_{12} &\in \mathbf{R}^{m(s+1)\times(m(s+1)-n)} \\ U_{21} &\in \mathbf{R}^{k_u(s+1)\times(k_u(s+1)+n)} \\ U_{22} &\in \mathbf{R}^{k_u(s+1)\times(m(s+1)-n)} \\ S &\in \mathbf{R}^{(k_u(s+1)+n)\times(k_u(s+1)+n)} \end{aligned}$$

and U, V are orthogonal matrices.

From

$$\begin{aligned} &\begin{bmatrix} U'_{12} & U'_{22} \end{bmatrix} W_N = 0 \\ &\Rightarrow \begin{bmatrix} U'_{12} & U'_{22} \end{bmatrix} \begin{bmatrix} H_{o\tau} & H_{s\tau} \\ O & I \end{bmatrix} \begin{bmatrix} X_N \\ U_N \end{bmatrix} = 0 \\ &\Rightarrow \begin{bmatrix} U'_{12} & U'_{22} \end{bmatrix} \begin{bmatrix} H_{o\tau} & H_{s\tau} \\ O & I \end{bmatrix} = 0 \end{aligned}$$

it follows that

$$\begin{cases} U'_{12}H_{o\tau} = 0 \\ U'_{12}H_{s\tau} + U'_{22} = 0. \end{cases}$$

As a result, matrices $H_{o\tau}^{\perp}$, $H_{o\tau}^{\perp}H_{s\tau}$ to be identified are delivered by

$$\begin{aligned} H_{o\tau}^{\perp} &= U'_{12} \\ H_{o\tau}^{\perp}H_{s\tau} &= -U'_{22}. \end{aligned} \quad (25)$$

It is worth noting that to ensure the existence of the left null space of W_N and H, the integer s should be chosen to be big enough. On the other side, the bigger s is, the more is the computational effort. If the bound of the system order is known a priori, then this information can be used to assist the selection of a suitable value of s.

3.3 Computation of $H_o^{\perp}$ and $H_o^{\perp}H_s$

If matrices $H_o^{\perp}, H_o^{\perp}H_s$ can be extracted from matrices $H_{o\tau}^{\perp}, H_{o\tau}^{\perp}H_{s\tau}$, then an observer-based residual generator can be readily obtained based on Lemma 1.

To figure out the relation between $H_{o\tau}^{\perp}, H_{o\tau}^{\perp}H_{s\tau}$ and $H_o^{\perp}, H_o^{\perp}H_s$, we note that

$$H_{o\tau} = \begin{bmatrix} C \\ C+\tau CA \\ \vdots \\ C_s^0C + C_s^1\tau CA + \cdots + C_s^s\tau^sCA^s \end{bmatrix} = \begin{bmatrix} I & O & \cdots & O \\ C_1^0I & C_1^1\tau I & \ddots & \vdots \\ \vdots & \ddots & \ddots & O \\ C_s^0I & C_s^1\tau I & \cdots & C_s^s\tau^sI \end{bmatrix} \begin{bmatrix} C \\ CA \\ \vdots \\ CA^s \end{bmatrix}$$

where C_i^j denotes the binomial coefficient of i and j. Define

$$P_m = \begin{bmatrix} I_{m\times m} & O & \cdots & O \\ C_1^0I_{m\times m} & C_1^1\tau I_{m\times m} & \ddots & \vdots \\ \vdots & \ddots & \ddots & O \\ C_s^0I_{m\times m} & C_s^1\tau I_{m\times m} & \cdots & C_s^s\tau^sI_{m\times m} \end{bmatrix}$$

Then

$$H_{o\tau} = P_mH_o. \quad (26)$$

Introduce another matrix

$$P_{k_u} = \begin{bmatrix} I_{k_u\times k_u} & O & \cdots & O \\ C_1^0I_{k_u\times k_u} & C_1^1\tau I_{k_u\times k_u} & \ddots & \vdots \\ \vdots & \ddots & \ddots & O \\ C_s^0I_{k_u\times k_u} & C_s^1\tau I_{k_u\times k_u} & \cdots & C_s^s\tau^sI_{k_u\times k_u} \end{bmatrix}$$

There is

$$H_{s\tau}P_{k_u} = \begin{bmatrix} D & O & \cdots & O \\ \tau CB & D & \ddots & \vdots \\ \vdots & \ddots & \ddots & O \\ \tau C(\tau A+I)^{s-1}B & \cdots & \tau CB & D \end{bmatrix} \times$$

$$\begin{bmatrix} I_{k_u\times k_u} & O & \cdots & O \\ C_1^0 I_{k_u\times k_u} & C_1^1\tau I_{k_u\times k_u} & \ddots & \vdots \\ \vdots & \ddots & \ddots & O \\ C_s^0 I_{k_u\times k_u} & C_s^1\tau I_{k_u\times k_u} & \cdots & C_s^s\tau^s I_{k_u\times k_u} \end{bmatrix} =$$

$$\begin{bmatrix} D & O & \cdots & O \\ \tau CB+D & \tau D & \ddots & \vdots \\ \vdots & \ddots & \ddots & O \\ \tau C(\tau A+I)^{s-1}B+\cdots+D & \cdots & \tau^s CB+\tau^{s-1}C_s^{s-1}D & \tau^s D \end{bmatrix} =$$

$$\begin{bmatrix} I_{m\times m} & O & \cdots & O \\ C_1^0 I_{m\times m} & C_1^1\tau I_{m\times m} & \ddots & \vdots \\ \vdots & \ddots & \ddots & O \\ C_s^0 I_{m\times m} & C_s^1\tau I_{m\times m} & \cdots & C_s^s\tau^s I_{m\times m} \end{bmatrix} \times$$

$$\begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ CA^{s-1}B & \cdots & CB & D \end{bmatrix} =$$

$$P_m H_s.$$

As a result, we get the following lemma.

Lemma 2. Matrices H_o and H_s defined by (5) and matrices $H_{o\tau}$ and $H_{s\tau}$ defined by (19) are related by

$$H_o^\perp = H_{o\tau}^\perp P_m \tag{27}$$

$$H_o^\perp H_s = H_{o\tau}^\perp H_{s\tau}P_{k_u} \tag{28}$$

where

$$P_j = \begin{bmatrix} I_{j\times j} & O & \cdots & O \\ C_1^0 I_{j\times j} & C_1^1\tau I_{j\times j} & \ddots & \vdots \\ \vdots & \ddots & \ddots & O \\ C_s^0 I_{j\times j} & C_s^1\tau I_{j\times j} & \cdots & C_s^s\tau^s I_{j\times j} \end{bmatrix} \tag{29}$$

$$j = m, k_u.$$

Proof. First, $H_{o\tau}^\perp H_{o\tau} = 0$ means that $H_{o\tau}^\perp P_m H_o = 0$ due to (26). Therefore, (27) is obtained. From $H_{s\tau}P_{k_u} = P_m H_s$ it follows that $H_{o\tau}^\perp H_{s\tau}P_{k_u} = H_{o\tau}^\perp P_m H_s = H_o^\perp H_s$, i.e. equation (28).

According to Lemma 2, if $H_{o\tau}^\perp$ and $H_{o\tau}^\perp H_{s\tau}$ are identified, then $H_o^\perp$ and $H_o^\perp H_s$ can be obtained, respectively, from (27) and (28). So one vector v_s satisfying (6) and the corresponding vector $\rho_s = v_s H_s$ can be chosen as

$$v_s = \alpha H_o^\perp, \quad \rho_s = \alpha H_o^\perp H_s \tag{30}$$

where α is any nonzero row vector of compatible dimensions. Substituting (25), (27) and (28) into (30), finally we get

$$\begin{aligned} v_s &= \alpha U_{12}' P_m \\ \rho_s &= -\alpha U_{22}' P_{k_u}. \end{aligned} \tag{31}$$

Applying Lemma 1, a residual generator in the form of (2) with coefficient matrices determined by (9) can then be constructed, whose residual dynamics is characterized by (4) and can be used for the aim of fault detection.

3.4 Design procedure

In summary, an observer-based residual generator in the form of

$$\begin{aligned} \dot{z}(t) &= Gz(t) + Ju(t) + Ly(t) \\ r(t) &= wz(t) + pu(t) + vy(t) \end{aligned} \tag{32}$$

can be designed for continuous LTI system (1), whose model is unknown, based on time domain data $y(t)$, $u(t)$ in normal operations as below:

1) Set the value of $\tau > 0$, N and s
2) Filter input and output signals $u(t), y(t)$ as shown in Fig. 1 to get $u^{(i)}(t)$, $y^{(i)}(t)$, $i = 1, \cdots, s$
3) Take samples of $u(t_k)$, $y(t_k)$, $u^{(i)}(t_k)$, $y^{(i)}(t_k)$, $k = 1, \cdots, N$
4) Build the data matrix W_N by (22) and (23)
5) Do the SVD of W_N as (24) to get matrix U and then partition U to get U_{12} and U_{22}
6) Calculate matrices P_m, P_{k_u} by (29)
7) Select a nonzero vector α and compute vectors v_s, ρ_s by (31)
8) Choose $g_1, g_2, \cdots, g_s$ so that all eigenvalues of G are on the left complex plane
9) Compute G, J, L, w, p, v according to (9)

4 Conclusion

In this paper, a model-free approach is presented to design an observer-based fault detection system of linear continuous-time systems based on input and output data in the time domain. The core of the approach is to directly identify parameters of the observer-based residual generator based on a numerically reliable data equation obtained by filtering and sampling the input and output signals. Current work is on the topic of a suitable handling of noises and disturbances.

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