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A Robust Fault Detection Approach for Nonlinear Systems

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Abstract: In this paper, we study the robust fault detection problem of nonlinear systems. Based on the Lyapunov method, a robust fault detection approach for a general class of nonlinear systems is proposed. A nonlinear observer is first provided, and a sufficient condition is given to make the observer locally stable. Then, a practical algorithm is presented to facilitate the realization of the proposed observer for robust fault detection. Finally, a numerical example is provided to show the effectiveness of the proposed approach.

Keywords: Robust, nonlinear systems, fault detection, observer, stability.

1 Introduction

Since, in practice, it is usually difficult to model a complex industrial process, the issue of robustness has been intensively studied in the past decade in the fault detection field and many approaches proposed.

As far as linear systems are concerned, Chen *et al.*[1] presented a robust fault detection filter, by combining an unknown input observer, and a fault detection filter, to produce a full-order unknown input observer. Yang and Saif[2] produced a reduced-order observer for fault detection in time delay systems with input uncertainties, and its stability and convergence were proved. Hamelin and Sauter[3], proposed an approach for the design of an optimal filter used for fault detection, by using a frequency domain technique useful for dealing with bounded uncertainties in linear systems.

Bilinear systems have had particular attention in recent years. Yu and Shields[4], proposed a bilinear fault detection filter (BFDF), and gave a sufficient condition for the existence of this BFDF. Jiang *et al.*[5], discussed the fault diagnosis problem in a class of uncertain bilinear system, where a state transformation technique was adopted to transform the original system into two subsystems; in which one was affected by an actuator fault, while the other was not.

For general nonlinear systems, Soroush[6], presented a model based fault diagnosis method; in which noisy data and modeling uncertainties could be dealt with. In systems in which states were not totally observable and there were modeling uncertainties, Wang *et al.*[7], proposed a fault diagnosis algorithm the main idea of which was to use a nonlinear estimator to monitor the system. Moreover, they provided a nonlinear estimation model, and a learning algorithm, such that fault amplitude could be estimated. Zhang *et al.*[8], considered a class of nonlinear system in which fault related nonlinearity is partially known. They modeled uncertainties, and proposed a robust fault diagnosis method.

The aim of this paper is to study the robust fault detection problem of a general class of nonlinear system. Based on the Lyapunov method, a robust fault detection approach to a general class of nonlinear system is proposed. First, a nonlinear observer is provided and a sufficient condition is given to make the observer locally stable. Then, a practical algorithm is presented to facilitate the realization of the proposed observer for robust fault detection. Finally, a numerical example is provided to show the effectiveness of the proposed approach.

2 Lyapunov method based robust fault detection

Similar to robust control, the Lyapunov method can be used to analyze the robust fault detection problem.

Consider the following nonlinear systems:

$$\begin{cases} \dot{\boldsymbol{x}} = f(\boldsymbol{x}, \boldsymbol{u}) + g(\boldsymbol{x})\boldsymbol{w} \\ \boldsymbol{y} = h(\boldsymbol{x}) \end{cases} \tag{1}$$

where state $\boldsymbol{x} \in R^n$, system input $\boldsymbol{u} \in R^p$, system disturbance $\boldsymbol{w} \in R^l$, system output $y \in R^m$, and non-

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linear continuous functions are $f : R^n \times R^p \to R^n$, $g : R^n \to R^n \times R^l$, and $h : R^n \to R^m$.

Let $\hat{\boldsymbol{x}}$ represent a state estimate, then a state observer can be constructed as:

$$\begin{cases} \dot{\hat{\boldsymbol{x}}} = f(\hat{\boldsymbol{x}}, \boldsymbol{u}) + L(\hat{\boldsymbol{x}}, \boldsymbol{u})[\boldsymbol{y} - h(\hat{\boldsymbol{x}})] \\ \hat{\boldsymbol{y}} = h(\hat{\boldsymbol{x}}) \end{cases} \tag{2}$$

where $L(\hat{\boldsymbol{x}}, \boldsymbol{u})$ is an observer gain matrix. Let $\boldsymbol{\varepsilon} = \boldsymbol{x} - \hat{\boldsymbol{x}}$ denote state estimation error, then we have the following error dynamic equation:

$$\begin{cases} \dot{\boldsymbol{\varepsilon}} = \tilde{f}(\hat{\boldsymbol{x}}, \boldsymbol{\varepsilon}, \boldsymbol{u} + g(\hat{\boldsymbol{x}} + \boldsymbol{\varepsilon})\boldsymbol{w} \\ \boldsymbol{\eta} = \tilde{h}(\hat{\boldsymbol{x}}, \boldsymbol{\varepsilon}) \end{cases} \tag{3}$$

where:

$$\tilde{f}(\hat{\boldsymbol{x}}, \boldsymbol{\varepsilon}, \boldsymbol{u}) = f(\hat{\boldsymbol{x}} + \boldsymbol{\varepsilon}, \boldsymbol{u}) - f(\hat{\boldsymbol{x}}, \boldsymbol{u}) - L(\hat{\boldsymbol{x}}, \boldsymbol{u})[h(\hat{\boldsymbol{x}} + \boldsymbol{\varepsilon}) - h(\hat{\boldsymbol{x}})] \tag{4}$$

$$\tilde{h}(\hat{\boldsymbol{x}}, \boldsymbol{\varepsilon}) = h(\hat{\boldsymbol{x}} + \boldsymbol{\varepsilon}) - h(\hat{\boldsymbol{x}}). \tag{5}$$

The problem now is to determine the gain matrix $L(\hat{\boldsymbol{x}}, \boldsymbol{u})$ such that the observer is stable in the presence of model uncertainty $\boldsymbol{w}$. To this end, we first introduce some concepts and lemmas.

Definition 1. System (1) is called zero-state detectable, if any of the solution $\boldsymbol{x}(t)$, under $\boldsymbol{y}(t) \equiv 0$ and $\boldsymbol{w} \equiv 0$, tends to zero.

Definition 2. Consider the system:

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}). \tag{6}$$

A set M is called an invariant-set of system (6), if $\boldsymbol{x}(t) \in M, \forall t \geqslant 0$ for any initial states $\boldsymbol{x}(0) \in M$.

Lemma 1. (local invariant-set theorem)[9] For system (6), if there is a first-order continuously differential function $V(\boldsymbol{x}) \in R$, satisfying:

1) there is a proper positive constant l, such that $\Omega_l = \{\boldsymbol{x} | V(\boldsymbol{x}) \leqslant l\}$ is bounded;

2) $\dot{V}(\boldsymbol{x}) \leqslant 0, \ \forall x \in \Omega_l$.

For any initial state $\boldsymbol{x}(0) \in \Omega_l$, the state trajectory $\boldsymbol{x}(t)$ will tend to the maximal invariant-set M in $R = \{\boldsymbol{x} | \dot{V}(\boldsymbol{x}) = 0, \boldsymbol{x} \in \Omega_l\}$ when $t \to \infty$, where the maximal invariant-set is defined to be the direct sum of all invariant-sets.

Lemma 2. (Global invariant-set theorem)[9] For system (6), if there exists a first-order continuously differential function $V(\boldsymbol{x}) \in R$, satisfying:

1) $V(\boldsymbol{x})$ is radially unbounded, i.e. $V(\boldsymbol{x}) \to \infty$ when $\|\boldsymbol{x}\| \to \infty$;

2) $\dot{V}(\boldsymbol{x}) \leqslant 0, \ \forall x$.

For any initial state $\boldsymbol{x}(0)$, the state trajectory $\boldsymbol{x}(t)$ will tend to the maximal invariant-set M in $R = \{\boldsymbol{x} | \dot{V}(\boldsymbol{x}) = 0\}$ when $t \to \infty$.

Theorem 1. Consider the error system (3), for a given $\gamma > 0$, if there exists a semi-positive definite function $V(\boldsymbol{\varepsilon}) \geqslant 0$, $\forall \boldsymbol{\varepsilon} \in R^n$, and matrix $L(\hat{\boldsymbol{x}}, \boldsymbol{u})$, such that the following Hamilton-Jacobi inequality:

$$\frac{\partial V}{\partial \boldsymbol{\varepsilon}} \tilde{f}(\hat{\boldsymbol{x}}, \boldsymbol{\varepsilon}, \boldsymbol{u}) + \frac{1}{2\gamma^2} \frac{\partial V}{\partial \boldsymbol{\varepsilon}} g(\hat{\boldsymbol{x}} + \boldsymbol{\varepsilon}) g^{\mathrm{T}}(\hat{\boldsymbol{x}} + \boldsymbol{\varepsilon}) \frac{\partial V^{\mathrm{T}}}{\partial \boldsymbol{\varepsilon}} + \frac{1}{2} \tilde{h}^{\mathrm{T}}(\hat{\boldsymbol{x}}, \boldsymbol{\varepsilon}) \tilde{h}(\hat{\boldsymbol{x}}, \boldsymbol{\varepsilon}) \leqslant 0 \tag{7}$$

exists, then:

$$\|\boldsymbol{\eta}\|_T \leqslant \gamma \|\boldsymbol{w}\|_T, \quad \forall \boldsymbol{w} \in L_2[0, T] \tag{8}$$

holds, for any given $T \geqslant 0$.

Proof. From (3) and (7), we have:

$$\frac{\partial V}{\partial \boldsymbol{\varepsilon}} \tilde{f}(\hat{\boldsymbol{x}}, \boldsymbol{\varepsilon}, \boldsymbol{u}) + \frac{\partial V}{\partial \boldsymbol{\varepsilon}} g(\hat{\boldsymbol{x}} + \boldsymbol{\varepsilon}) \boldsymbol{w} \leqslant \frac{\partial V}{\partial \boldsymbol{\varepsilon}} g(\hat{\boldsymbol{x}} + \boldsymbol{\varepsilon}) \boldsymbol{w} - \frac{1}{2\gamma^2} \frac{\partial V}{\partial \boldsymbol{\varepsilon}} g(\hat{\boldsymbol{x}} + \boldsymbol{\varepsilon}) g^{\mathrm{T}}(\hat{\boldsymbol{x}} + \boldsymbol{\varepsilon}) \frac{\partial V^{\mathrm{T}}}{\partial \boldsymbol{\varepsilon}} - \frac{1}{2} \tilde{h}^{\mathrm{T}}(\hat{\boldsymbol{x}}, \boldsymbol{\varepsilon}) \tilde{h}(\hat{\boldsymbol{x}}, \boldsymbol{\varepsilon}) - \frac{1}{2}\gamma^2 \boldsymbol{w}^{\mathrm{T}} \boldsymbol{w} + \frac{1}{2}\gamma^2 \boldsymbol{w}^{\mathrm{T}} \boldsymbol{w} \tag{9}$$

that is,

$$\dot{V} \leqslant \frac{1}{2}\{\gamma^2 \|\boldsymbol{w}\|^2 - \|\boldsymbol{\eta}\|^2\} - \frac{\gamma^2}{2} \left\| \boldsymbol{w} - \frac{1}{\gamma^2} g^{\mathrm{T}}(\hat{\boldsymbol{x}} + \boldsymbol{\varepsilon}) \frac{\partial V^{\mathrm{T}}}{\partial \boldsymbol{\varepsilon}} \right\|^2 \tag{10}$$

hence,

$$\dot{V} \leqslant \frac{1}{2}\{\gamma^2 \|\boldsymbol{w}\|^2 - \|\boldsymbol{\eta}\|^2\}. \tag{11}$$

By integration, this leads to:

$$V(\boldsymbol{\varepsilon}(T)) - V(\boldsymbol{\varepsilon}(0)) \leqslant \frac{1}{2} \int_0^T \gamma^2 \|\boldsymbol{w}\|^2 dt - \frac{1}{2} \int_0^T \|\boldsymbol{\eta}\|^2 dt. \tag{12}$$

Since $V(\boldsymbol{\varepsilon}(T)) \geqslant 0$ and $V(\boldsymbol{\varepsilon}(0)) = 0$, therefore, we have:

$$\int_0^T \|\boldsymbol{\eta}\|^2 dt \leqslant \gamma^2 \int_0^T \|\boldsymbol{w}\|^2 dt, \tag{13}$$

i.e.

$$\|\boldsymbol{\eta}\|_T^2 \leqslant \gamma^2 \|\boldsymbol{w}\|_T^2, \quad \forall \boldsymbol{w} \in L_2[0, T] \tag{14}$$

which completes the proof. □

Theorem 1 gives only a sufficient condition to make the system gain less than a given value. The following theorem will further discuss the stability problem when $\boldsymbol{w} = 0$.

Theorem 2. Assume that system (3) is zero-state detectable. If there exists a semi-positive definite function $V(\boldsymbol{\varepsilon}) \geqslant 0$ and $\forall \boldsymbol{\varepsilon} \in R^n$ satisfying the Hamilton-Jacobi inequality (7), then the equilibrium point $\boldsymbol{x} = 0$ is asymptotically stable.

Proof. Since system (3) is zero-state detectable, the arbitrary solution $\varepsilon(t)$, satisfying:

$$\begin{cases} \dot{\varepsilon} = \tilde{f}(\hat{x}, \varepsilon, u) \\ \eta = \tilde{h}(\hat{x}, \varepsilon) = 0 \end{cases}$$

will tend to 0 , which implies that the set of $\varepsilon(t)$ is an invariant-set. Let $w = 0$, and (11) become:

$$\dot{V} \leqslant -\frac{1}{2}\|\eta\|^2. \tag{15}$$

Therefore, the set $R = \{x|\dot{V} = 0\}$ is an invariant-set. It follows from lemma 1 that $x(t)$ will tend to this invariant-set for any arbitrary initial states $x(0)$, that is, $x(t)$ will converge to 0 as $t \to \infty$. □

Although Theorems (1) and (2), give us the sufficient condition that a robust observer exists for nonlinear system (1), these theorems don't tell us how to construct observer gain. In fact, for general nonlinear systems, there has been no efficient algorithm to solve Hamilton-Jacobi inequality (7). In spite of this, we can use theorems (1) and (2), to test whether an observer has robustness against disturbance. In the case of linear systems, this problem can be solved completely.

Consider the following linear systems:

$$\begin{cases} \dot{x} = Ax + B_2u + B_1w \\ y = Cx \end{cases} \tag{16}$$

where $x \in R^n$, $u \in R^p$, $w \in R^l$, and $y \in R^m$; and A, B_1, B_2, and C are matrices with proper dimensions.

The observer is:

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + B_2u + L(y - \hat{y}) \\ \hat{y} = C\hat{x} \end{cases} \tag{17}$$

where L is the gain matrix, and the error equation is:

$$\begin{cases} \dot{\varepsilon} = (A - LC)\varepsilon + B_1w \\ \eta = C\varepsilon \end{cases}. \tag{18}$$

Therefore, we have $\tilde{f}(\hat{x}, \varepsilon, u) = (A - LC)\varepsilon$, $g(\hat{x} + \varepsilon) = B_1$, and $\tilde{h}(\hat{x}, \varepsilon) = C\varepsilon$. Now, we select a Lyapunov function:

$$V(\varepsilon) = \frac{1}{2}\varepsilon^{\mathrm{T}}P\varepsilon \tag{19}$$

where P is a positive definite matrix. The Hamilton-Jacobi inequality now has the following form:

$$\varepsilon^{\mathrm{T}}P(A - LC)\varepsilon + \frac{1}{2\gamma^2}\varepsilon^{\mathrm{T}}PB_1B_1^{\mathrm{T}}P\varepsilon + \frac{1}{2}\varepsilon^{\mathrm{T}}C^{\mathrm{T}}C\varepsilon \leqslant 0. \tag{20}$$

(20) is equivalent to the following Riccati equation:

$$P(A–LC)+(A–LC)^{\mathrm{T}}P+\frac{1}{\gamma^2}PB_1B_1^{\mathrm{T}}P+C^{\mathrm{T}}C \leqslant 0. \tag{21}$$

There have been many efficient methods to solve this equation[10], therefore, gain matrix L can easily be determined.

3 A practical algorithm to design a robust observer

In this section, we present a practical algorithm to design a gain matrix L, and apply the observer for robust fault detection.

First, we linearize system (3) at the equilibrium point $x = 0$, and obtain:

$$\dot{\varepsilon} = A_F\varepsilon + g(\hat{x})w \tag{22}$$

where:

$$A_F = \Big[\frac{\partial f(\hat{x} + \varepsilon, u)}{\nabla\varepsilon} - L(\hat{x}, u)\frac{\partial h(\hat{x} + \varepsilon)}{\partial\varepsilon} + \frac{\partial(g(\hat{x} + \varepsilon)w)}{\partial\varepsilon}\Big]\Big|_{\varepsilon=0}. \tag{23}$$

Then we use a heuristic method to find a proper $L(\hat{x}, u)$, such that all eigenvalues of A_F have a negative real part, which ensures error system (22) will be stable.

Theorem 3. Assume $\|w\| \leqslant \sigma$ and $\|g(\cdot)\| \leqslant \delta$, then error system (22) is locally stable, if A_F is a stable matrix.

Proof. Since A_F is a stable matrix, there therefore exists $\rho > 0$ and $\lambda > 0$ such that:

$$\|e^{A_Ft}\| \leqslant \rho e^{-\lambda t}. \tag{24}$$

The solution of (22) is:

$$\varepsilon(t) = \varepsilon_0 e^{A_Ft} + \int_0^t e^{A_F(t-\tau)}g(\hat{x})w d\tau. \tag{25}$$

It follows from (24) and (25) that:

$$\|\varepsilon(t)\| \leqslant \varepsilon_0\rho e^{-\lambda t} + \rho\int_0^t e^{-\lambda(t-\tau)}\|g[\hat{x}(\tau)]\| \cdot \|w\| d\tau. \tag{26}$$

With the assumption $g(\cdot)$, we finally obtain:

$$\|\varepsilon(t)\| \leqslant \varepsilon_0\rho + \rho\delta\sigma\frac{1 - e^{-\lambda t}}{\lambda} < \varepsilon_0\rho + \frac{\rho\delta\sigma}{\lambda}. \tag{27}$$

which shows that the linearized error equation is stable. This implies that original error equation (3) is locally stable. □

There exist similar results for discrete-time systems. Consider the following systems:

$$\begin{cases} x(k+1) = f(x(k), u(k)) + g(x(k))w(k) \\ y(k) = h(x(k)) \end{cases} \tag{28}$$

where x, u, w, y, f, g and h have the same meaning as in (1).

The observer equation for system (28) is:

$$\begin{cases} \hat{\boldsymbol{x}}(k+1) = f(\hat{\boldsymbol{x}}(k), \boldsymbol{u}(k)) \\ \qquad +L(\hat{\boldsymbol{x}}(k), \boldsymbol{u}(k))[h(\boldsymbol{x}(k)) - h(\hat{\boldsymbol{x}}(k))] \\ \hat{\boldsymbol{y}}(k) = h(\hat{\boldsymbol{x}}(k)) \end{cases} \tag{29}$$

with error equation:

$$\boldsymbol{\varepsilon}(k+1) = A_F(k)\boldsymbol{\varepsilon}(k) + g(\hat{\boldsymbol{x}}(k))\boldsymbol{w}(k). \tag{30}$$

For the purpose of fault detection, we should select an upper bound $\boldsymbol{\varepsilon}^+(k)$, and a lower bound $\boldsymbol{\varepsilon}^-(k)$, of the estimation error $\boldsymbol{\varepsilon}(t)$, which can be determined by:

$$\boldsymbol{\varepsilon}^+(k+1) = A_F(k)\boldsymbol{\varepsilon}(k) + \sup(g(\hat{\boldsymbol{x}}(k)) \cdot \boldsymbol{w}) \tag{31}$$

$$\boldsymbol{\varepsilon}^-(k+1) = A_F(k)\boldsymbol{\varepsilon}(k) + \inf(g(\hat{\boldsymbol{x}}(k)) \cdot \boldsymbol{w}) \tag{32}$$

where:

$$\begin{aligned} A_F(k) =& \Big[\frac{\partial f(\hat{x}(k)+\varepsilon(k), u(k))}{\partial \varepsilon(k)} \\ & - L(\hat{x}(k), u(k))\frac{\partial h(\hat{x}(k)+\varepsilon(k))}{\partial \varepsilon(k)} \\ & + \frac{\partial g(\hat{x}(k)+\varepsilon(k))w(k)}{\partial \varepsilon(k)}\Big]\Big|_{\varepsilon(k)=0}. \end{aligned} \tag{33}$$

When $\varepsilon(k)$ is outside $[\boldsymbol{\varepsilon}^-(k), \boldsymbol{\varepsilon}^+(k)]$, we will conclude that there has been a fault in the system.

In practice, we can first design $L(\hat{\boldsymbol{x}}(k), \boldsymbol{u}(k))$ by assigning eigenvalues of $A_F(k)$ in the unit circle; and then calculate $\boldsymbol{\varepsilon}^+(k)$ and $\boldsymbol{\varepsilon}^-(k)$ by (31) and (32). The reason we don't use (27) to calculate the upper and lower bound of estimation error, is that (27) usually gives larger bounds, which are harmful when detecting incipient faults.

4 Simulation studies

Consider the following well-known Van der Pol oscillator:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ 2\omega\zeta(1-\alpha x_1^2)x_2 - \omega^2 x_1 \end{bmatrix} + \begin{bmatrix} 0 \\ u \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \tag{34}$$

$$y = x_1 \tag{35}$$

where w_1 and w_2 are model uncertainties, or system disturbance, and ω, ζ and α are positive constants.

With sampling period T, the system can be discretized using a Eular method, which leads to:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} x_1(k) + Tx_2(k) \\ x_2(k) + 2T\omega\zeta(1-\alpha x_1^2(k))x_2(k) - T\omega^2 x_1(k) \end{bmatrix} + \begin{bmatrix} 0 \\ Tu(k) \end{bmatrix} + T \cdot \begin{bmatrix} w_1(k) \\ w_2(k) \end{bmatrix} \tag{36}$$

$$y(k) = x_1(k). \tag{37}$$

In order to generate the eigenvalues of A_F in the disc, we obtain a suitable $L(\hat{\boldsymbol{x}}(k), \boldsymbol{u}(k)) = [L_1\ L_2]^{\mathrm{T}}$:

$$L_1 = a_0 + 2 + 2T\omega\zeta(1-\alpha\hat{x}_1^2(k)) \tag{38}$$

$$\begin{aligned} L_2 =& b_0 + \frac{1}{T}(L_1 - 1)[1 + 2T\omega\zeta(1-\alpha\hat{x}_1^2(k))] \\ & - 4T\omega\zeta\alpha\hat{x}_1(k)\hat{x}_2(k) - T\omega^2 \end{aligned} \tag{39}$$

where a_0 and b_0 are parameters to be estimated.

If the eigenvalues of A_F are $(-a_0 \pm \sqrt{a_0^2 - 4b_0T})/2$, we should select properly a_0 and b_0 such that:

$$|(-a_0 \pm \sqrt{a_0^2 - 4b_0T})/2| < 1. \tag{40}$$

In simulations, we select: $\omega = 0.9$, $\zeta = 0.6$, $\alpha = 0.95$, $T = 0.1$, $a_0 = 1$ and $b_0 = 2$. System input is $0.1\sin(t/T)$, and we model uncertainties as $|w_1|, |w_2| \leqslant 0.1$. At each step between $k = 150$ and 160, we add a constant additive fault into the first state equation in (36), with an amplitude of +0.05. Simulation results are shown in Fig.1.

Fig.2 shows the simulation results when x_2 can also be measured, which shows better performance than in Fig.1. This is due to the fact that less cumulative error of the observer occurs, when the observability of the system is improved after adding a new measurement.

The simulation results in Fig.1 show that it is difficult to detect a fault just from output. With our method at $k = 152$, we detect a fault, since at this moment real estimation error ε_1 exceeds the upper bound ε_1^+.

5 Conclusions

In robust fault detection, there exist two contradictory requirements, i.e. 1) robustness against model uncertainties, noise and various disturbance is required, while 2) sensitivity is required to system faults. Therefore, one has to make a compromise between both requirements. This paper discusses the robust fault detection problem for nonlinear systems over a time domain. On the basis of the Lyapunov method, we provide a sufficient condition for the existence of a robust observer. As long as we can find an efficient algorithm to solve the Hamilton-Jacobi inequality we can obtain the robust observer, and carry out robust fault detection. The proposed approach has wide application, since there are few limitations on the system model. The practical algorithm given in Section 3 is a local, robust fault detection method, which can also be applied to time-varying nonlinear systems.

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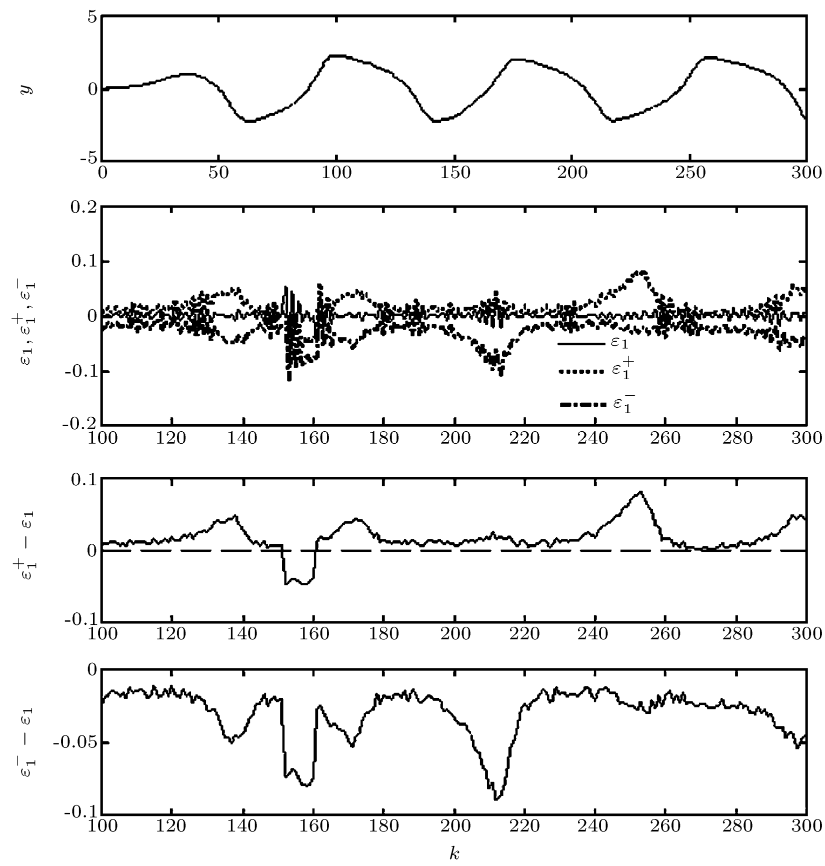

Fig.1 Simulation results when x_1 is measured

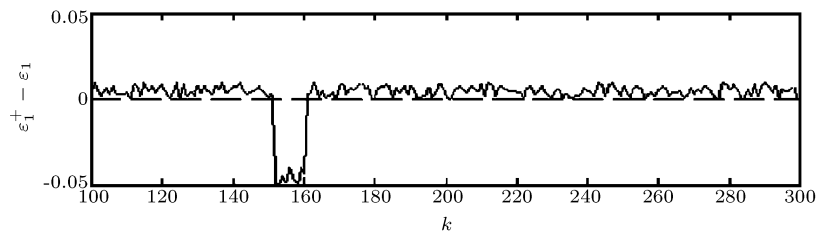

Fig.2 Simulation results when all states can be measured

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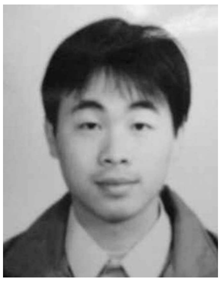

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