

# Sliding Mode Guidance Law Considering Missile Dynamic Characteristics and Impact Angle Constraints

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**Abstract:** In order to improve the precision of guidance for the missile intercepting maneuvering targets, this paper proposes a sliding mode guidance law with impact angle constraints based on the equation of the relative motion of the missile and the target in a 2D plane. Two finite-time convergent guidance laws are proposed based on the nonsingular terminal sliding mode, while, two exponential convergent guidance laws involving dynamic delay are developed through applying the higher-order nonsingular terminal sliding mode. The simulations denote that, in all the four scenarios of the target's maneuvering, the guidance laws are able to inhibit the chattering phenomenon of the sliding modes effectively; and from an expected aspect angle, the missiles could attack the targets with high precision and fast speed.

**Keywords:** Autopilot, impact angle, nonsingular terminal sliding mode, finite-time convergent, guidance

## 1 Introduction

With the continuous development of precision-guided munitions, the guidance law design is constrained not only by the requirement on miss distances, but also by the requirement on the angle of the guidance terminal phase in many scenarios, so as to allow the maximum effectiveness of the warhead, and to achieve the best damage effect. For this reason, it is necessary to do further study on the guidance law with terminal impact angle constraints, to meet the requirement of this special guidance mission.

In 1973, Kim and Grider proposed an optimal guidance law with impact angle control for the reentry vehicle in the vertical plane based on a linear model<sup>[1]</sup>. Since then, various robust guidance laws with impact angle constraints have been proposed for different scenarios. Chai et al.<sup>[2]</sup> reviewed the domestic and international researches on guidance laws with terminal angle constraints, and analyzed both the advantages and disadvantages of the optimal guidance law, variable structure guidance law, improved proportional navigation guidance law and integrated guidance law. It points out the importance of guidance law with terminal angle constraints in improving the combat effectiveness of guided munitions; while there are still practical problems to be effectively addressed despite of the great progress in the research on robust guidance law. Ratnoo and Ghose<sup>[3]</sup> employed the traditional proportional navigation guidance law to discuss the conditions to choose all the navigation coefficients, and it concluded that during ground-to-ground

attack, it is available to attack with desired impact angle constraints and small miss distances but it applies to only stationary targets. Shima<sup>[4]</sup> analyzed the relationship between the speed ratio and the lead angle based on the three possible modes of the collision between missiles and targets, employed sliding mode control to design guidance law, which is then compared with the proportional navigation guidance law, and simulated several maneuvering profiles of the targets. Harl and Balakrishnan<sup>[5]</sup> employed the theory of second-order sliding mode to design the guidance law satisfying the desired line-of-sight (LOS) angle curve and scheduled attack time, and applied this guidance law to multi-missile salvo attack. However, this method does not consider the dynamic delay of missile autopilot, and is only applicable to stationary or slow targets. Shashi et al.<sup>[6]</sup> employed the traditional terminal sliding mode to design a guidance law with impact angle constraints, which is then applied to attack the stationary targets, as well as interception of constant speed and maneuvering targets. However, it does not consider the issue of singularity of the traditional sliding mode. Sachit and Debasish<sup>[7]</sup> employed the theory of variable structure to study the guidance law with impact angle constraints by switching among different sliding mode controls, and analyzed the capability of missile to capture maneuvering targets. Sun et al.<sup>[8]</sup> combined sliding mode and backstepping theory to study the guidance law with impact angle constraint when there is dynamic delay of missile autopilot, but backstepping calls for a quite huge computation amount.

Due to the preferable adaptivity and robustness of sliding mode variable structure control to parameter perturbation and external disturbance, it has been widely applied in missile guidance and control, and it is obvious that it is applied by all the literatures mentioned above. However, it has been

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the objective of the study of control system to remove the chattering behavior of sliding mode, and to improve the speed to reach the sliding mode manifold. Gao<sup>[9]</sup> presented several concepts to improve the speed to reach the sliding mode manifold, including constant speed reaching law, exponential reaching law and power reaching law. However, the speed of constant speed reaching law is too slow; the convergence rate of exponential reaching law is quite fast, while there is greater system chattering when the system state is near sliding mode; power reaching law is conducive to reduce chattering, while the speed of the reaching stage is too slow when the system state is far from sliding mode, which calls for too much time<sup>[10]</sup>. Yu et al.<sup>[11]</sup> combined the traditional power reaching law and exponential reaching law to get a fast power reaching law, which can weaken the chattering of sliding mode, and improve the speed when the system state is far from the sliding mode manifold.

The traditional sliding mode control is designed with linear sliding mode manifold, and when the system state has reached to the sliding mode manifold, it will asymptotically converge to the equilibrium of the system, while is not finite-time convergent<sup>[7-9]</sup>. The terminal sliding mode control realizes the finite-time convergence of the system state, with a convergence performance better than that of the traditional sliding mode control<sup>[12, 13]</sup>. However, there is still the issue of singularity of the terminal sliding mode control, for which Feng et al.<sup>[14]</sup> presented a nonsingular terminal sliding mode control that overcomes the issue of singularity, to improve control performance.

The dynamic delay of missile autopilot is a main factor influencing the precision of guidance, and it is of practical engineering significance to consider the missile's dynamic characteristics in actual process of guidance. Sun et al.<sup>[15-17]</sup> designed a guidance law considering the missile's dynamic characteristics by a method which is still based on backstepping, while with no consideration of impact angle constraints. The main objective of this paper is to design a new guidance method which considers both the dynamic delay of missile autopilot and impact angle constraints. By applying the theory of nonsingular terminal sliding mode, rapid power reaching law and exponential reaching law, firstly, the guidance law with impact angle constraints is designed without the dynamic delay of autopilot. Furthermore, the guidance law with impact angle constraints is extended to the case of dynamic delay of autopilot. The numerical simulation is performed for different maneuvers manners of the target, which verifies the effectiveness of the guidance law designed in this paper.

## 2 Problem formulation

Considering the relative motion of the missile and the target in the intercepting plane  $oxy$ , both of which are regarded as point masses, and their connecting line is the LOS, as shown in Fig. 1.

In Fig. 1, The missile is denoted as  $M$  and the target is denoted as  $T$ ,  $r$  is the relative distance between the missile

and the target,  $\dot{r}$  is the derivative of  $r$  with respect to time,  $V_t, V_m$  the target's and the missile's speeds respectively, which are assumed to be constants.  $q$  is the LOS angle,  $\dot{q}$  is the derivative of  $q$  with respect to time,  $\phi_t, \phi_m$  the flight path angle of target and missile, respectively. Then differential equations can be derived from Fig. 1 as

$$\dot{r} = V_t \cos(q - \phi_t) - V_m \cos(q - \phi_m) \tag{1}$$

$$r\dot{q} = -V_t \sin(q - \phi_t) + V_m \sin(q - \phi_m). \tag{2}$$

Taking the derivative of (2), we can get

$$\ddot{q} = -\frac{2\dot{r}}{r}\dot{q} - \frac{1}{r}a_m + \frac{1}{r}a_t \tag{3}$$

where  $a_m = V_m\dot{\phi}_m(t) \cos(q - \phi_m)$ ,  $a_t = V_t\dot{\phi}_t(t) \cos(q - \phi_t)$  are the components of the acceleration of the missile and target normal to the LOS. According to the principle of quasi-parallel approaching method, the key to guidance law design is to control the LOS angle rate  $\dot{q}$  by  $a_m$ , and let it tend to zero, to ensure hitting the target precisely.

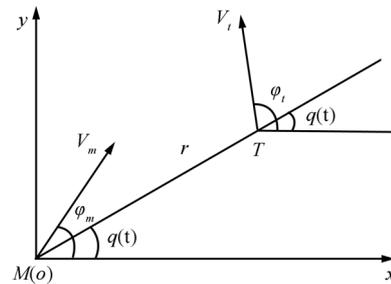


Fig. 1 Relative motion geometry of missile and target

The dynamics of the missile autopilot is described by the following first-order term:

$$\dot{a}_m = -\frac{1}{\tau}a_m + \frac{1}{\tau}u \tag{4}$$

where  $\tau$  is the time constant of missile autopilot,  $u$  the guidance command acceleration given to missile autopilot, and  $a_m$  the missile's acceleration obtained.

The impact angle is the included angle between the missile's speed vector and the target's speed vector in the critical terminal phase of the guidance. With the terminal time of guidance to be defined as  $t_f$ , and the missile's expected impact angle as  $\phi_0$ , the issue of guidance with impact angle control is to ensure a miss distance of zero and hitting the target from the expected impact angle at the terminal time of guidance, which means

$$\lim_{t \rightarrow t_f} r(t)\dot{q}(t) = 0 \tag{5}$$

$$\phi_m(t_f) - \phi_t(t_f) = \phi_0 \tag{6}$$

$$|q(t_f) - \phi_m(t_f)| < \frac{\pi}{2}. \tag{7}$$

Equation (7) means that the target is in the vision scope when the missile hits it, and we can get from (2) and (5) that:

$$V_t \sin[q(t_f) - \phi_t(t_f)] = V_m \sin[q(t_f) - \phi_m(t_f)]. \quad (8)$$

For a missile with a specific attack mission, the expected impact angle  $\phi_0$  is a constant, and when  $\phi_t(t_f)$  is known, the missile's trajectory inclination angle  $\phi_m(t_f)$  will become the expected one  $\phi_d$ , and it is available to get the certain and unique expected terminal LOS angle by (7) and (8), which can be defined to be  $q_d$ , and then the issue of guidance with impact angle control becomes the issue of terminal LOS angle satisfying  $q(t_f) = q_d$ .

**Remark 1.** If the target is stationary, then  $\phi_t(t_f) = 0$ . For non-maneuvering target,  $\phi_t(t_f)$  is measurable, while, for the maneuvering target,  $\phi_t(t_f)$  can be obtained through the ground radar detection. Hence, we suppose that the value of  $\phi_t(t_f)$  is known in the process of guidance law design.

**Assumption 1.** As restrained by acceleration capability, the maximum lateral acceleration that can be actually provided by the missile and the target is limited, therefore there exists a constant  $A_m > 0, A_1 > 0, A_2 > 0$  which allows:

$$|a_m| \leq A_m, \quad |a_t| \leq A_1, \quad |\dot{a}_t| \leq A_2. \quad (9)$$

During terminal guidance, as restrained by the power of its angle tracing system, receiver acceleration and other factors, the seeker has a minimum operating range  $r_0$ . When the relative distance between the missile and the target is no more than  $r_0$ , the guidance circuit is broken, therefore the guidance process satisfies the hypothesis below<sup>[18]</sup>.

**Assumption 2.** The time-varying parameter  $r(t)$  in system (3) satisfies:

$$r(t) \geq r_0. \quad (10)$$

For convenience of guidance law design, some definitions and lemmas are presented as follows.

**Definition 1**<sup>[19]</sup>. Considering a nonlinear system

$$\dot{x} = f(x, t), \quad f(0, t) = 0, \quad x \in \mathbf{R}^n \quad (11)$$

where,  $f : U_0 \times \mathbf{R} \rightarrow \mathbf{R}^n$  is continuous on  $U_0 \times \mathbf{R}$ , and  $U_0$  is an open neighborhood with the origin  $x = 0$ . The system's equilibrium  $x = 0$  (local) is finite-time convergent, which means that for the given initial state  $x(t_0) = x_0 \in U_0$  at any initial time  $t_0$ , there is a down time depending on,  $x_0$  which makes the solution  $x(t) = \varphi(t; t_0, x_0)$  of (11) with initial state  $x_0$  to be defined (which might not be unique), and

$$\begin{cases} \lim_{t \rightarrow T(x_0)} \varphi(t; t_0, x_0) = 0 \\ \text{if } t > T(x_0), \text{ then } \varphi(t; t_0, x_0) = 0 \end{cases}$$

which means when  $t \in [t_0, T(x_0)]$ ,  $\varphi(t; t_0, x_0) \in U_0 \setminus \{0\}$ . Additionally, the system's equilibrium  $x = 0$  (local) is finite-time stable, which means that it is Lyapunov stable, and finite-time convergent within a neighborhood  $U \subset U_0$  of the

origin. If  $U = \mathbf{R}^n$ , the origin is the equilibrium which is globally finite-time stable.

**Lemma 1**<sup>[11]</sup>. Considering a nonlinear system of (11), if there is a continuous and positive definite function  $V(t)$  satisfying the differential inequality as following:

$$\dot{V}(x) + \mu V(x) + \lambda V^\alpha(x) \leq 0 \quad (12)$$

where  $\mu, \lambda > 0, 0 < \alpha < 1$  are all constants, the time  $T$  for the system state to reach the stable point satisfies the inequality below:

$$T \leq \frac{1}{\mu(1-\alpha)} \ln \frac{\mu V^{1-\alpha}(x_0) + \lambda}{\lambda}.$$

### 3 Guidance law design

#### 3.1 Guidance law design not considering the dynamic delay of missile autopilot

Select state variables

$$x_1 = q(t) - q_d, \quad x_2 = \dot{x}_1. \quad (13)$$

Taking the derivative of (13), we can get the state equation of guidance system with impact angle constraints:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{2\dot{r}}{r} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{r} \end{bmatrix} a_m + \begin{bmatrix} 0 \\ \frac{1}{r} \end{bmatrix} a_t. \quad (14)$$

Let  $g(t) = \frac{1}{r} a_t$ , and we can get from Assumptions 1 and 2 that

$$|g(t)| = \left| \frac{1}{r} a_t \right| \leq \frac{A_1}{r_0} = \delta. \quad (15)$$

For the system of (14), select the nonsingular terminal sliding mode manifold

$$s = x_1 + \beta \text{sig}^\gamma(x_2) \quad (16)$$

where  $\beta > 0, 1 < \gamma < 2, \text{sig}^\gamma(x_2) = |x_2|^\gamma \text{sgn}(x_2)$ .

When the target is not maneuvering,  $a_t = 0$ , we select rapid power reaching law

$$\dot{s} = -k_1 s - k_2 \text{sig}^\rho(s) \quad (17)$$

where  $k_1, k_2 > 0, 0 < \rho < 1$ .

Taking the derivative of (16), we can get

$$\begin{aligned} \dot{s} &= \dot{x}_1 + \beta \gamma |x_2|^{\gamma-1} \dot{x}_2 = \\ & x_2 + \beta \gamma |x_2|^{\gamma-1} \left( -\frac{2\dot{r}}{r} x_2 - \frac{1}{r} a_m + \frac{1}{r} a_t \right). \end{aligned} \quad (18)$$

We can get from (17) and (18) that

$$\begin{aligned} a_m &= r(\beta^{-1} \gamma^{-1} \text{sig}^{2-\gamma}(x_2) - \frac{2\dot{r}}{r} x_2 + \\ & \beta^{-1} \gamma^{-1} |x_2|^{1-\gamma} (k_1 s + k_2 \text{sig}^\rho(s))). \end{aligned} \quad (19)$$

As the factor  $\beta^{-1}\gamma^{-1}|x_2|^{1-\gamma}$  will cause singularity when  $x_2 \rightarrow 0$ , we can design the guidance law for the system of (14) as

$$\bar{a}_m = r(\beta^{-1}\gamma^{-1}\text{sig}^{2-\gamma}(x_2) - \frac{2\dot{r}}{r}x_2 + k_1s + k_2\text{sig}^\rho(s)). \tag{20}$$

**Theorem 1.** For the system of (14), when  $a_t = 0$ , the system can reach the sliding mode manifold in finite time under the guidance law of (20). On the sliding mode  $s = 0$ , the LOS angle will converge to the desired LOS angle in finite time, and the LOS angle rate will converge to zero in finite time.

**Proof.** When the guidance law of (20) is substituted into (18), we can get

$$\dot{s} = \beta\gamma|x_2|^{\gamma-1}(-k_1s - k_2\text{sig}^\rho(s)).$$

Construct Lyapunov function

$$V = s^2. \tag{21}$$

Taking the derivative of (21), we can get

$$\begin{aligned} \dot{V} &= 2s\dot{s} = \\ &2\beta\gamma|x_2|^{\gamma-1}s(-k_1s - k_2\text{sig}^\rho(s)) = \\ &2\beta\gamma|x_2|^{\gamma-1}(-k_1s^2 - k_2|s|^{\rho+1}) = \\ &-\mu V - \lambda V^{\frac{\rho+1}{2}} \leq 0 \end{aligned}$$

where  $\mu = 2k_1\beta\gamma|x_2|^{\gamma-1} \geq 0$ ,  $\lambda = 2k_2\beta\gamma|x_2|^{\gamma-1} \geq 0$ . When  $x_2 \neq 0$ , we can get from Lemma 1 that the system converges to the sliding mode manifold  $s = 0$  in finite time. When  $x_2 = 0$ ,  $s \neq 0$ , we can get from (16) that  $x_1 \neq 0$ , and the system have not reached the equilibrium, thus it would not stop at  $\dot{V} = 0$ . As  $\dot{V} \leq 0$ , we can get that the system converges to the nonsingular sliding mode manifold  $s = 0$  in finite time. On  $s = 0$ ,  $x_1, x_2$  converges to the equilibrium in finite time, which means that the LOS angle converges to the desired value, and the LOS angle rate converges to zero.  $\square$

When the target is maneuvering,  $|g(t)| = |\frac{1}{r}a_t| \leq \delta$ , we select the sliding mode manifold of (16) and exponential reaching law

$$\dot{s} = -k_3s - k_4\text{sgn}(s) \tag{22}$$

where  $k_3, k_4 > 0$ .

Taking the derivative of (16), we can get

$$\begin{aligned} \dot{s} &= \dot{x}_1 + \beta\gamma|x_2|^{\gamma-1}\dot{x}_2 = \\ &x_2 + \beta\gamma|x_2|^{\gamma-1}\left(-\frac{2\dot{r}}{r}x_2 - \frac{1}{r}a_m + \frac{1}{r}a_t\right). \end{aligned} \tag{23}$$

Similarly, in order to avoid singularity, we design the guidance law for the system of (14) as

$$\bar{a}_m = r(\beta^{-1}\gamma^{-1}\text{sig}^{2-\gamma}(x_2) - \frac{2\dot{r}}{r}x_2 + k_3s + \eta\text{sgn}(s)). \tag{24}$$

**Theorem 2.** For the guidance law of (24), if the term of variable structure selected is  $\eta \geq k_4 + \delta$ ,  $\delta = \text{const.} > 0$ ,

this guidance law can compensate the target's maneuvering in the guidance system of (14) effectively, and make the system to reach the sliding mode manifold in finite time. On the sliding mode  $s = 0$ , the LOS angle will converge to the desired LOS angle in finite time, and the LOS angle rate will converge to zero in finite time.

**Proof.** When the guidance law of (24) is substituted into (23), we can get

$$\dot{s} = \beta\gamma|x_2|^{\gamma-1}(-k_3s - \eta\text{sgn}(s) + g(t)).$$

Construct Lyapunov function

$$V_1 = s^2 \tag{25}$$

Taking the derivative of (25), we can get

$$\begin{aligned} \dot{V} &= 2s\dot{s} = \\ &2\beta\gamma|x_2|^{\gamma-1}s(-k_3s - \eta\text{sgn}(s) + g(t)) \leq \\ &2\beta\gamma|x_2|^{\gamma-1}(-k_3s^2 - \eta|s| + |g(t)s|) \leq \\ &2\beta\gamma|x_2|^{\gamma-1}(-k_3s^2 - (\eta - \delta)|s|) = \\ &-\mu_1V - \lambda_1V^{\frac{1}{2}} \leq 0 \end{aligned}$$

where  $\mu_1 = 2k_3\beta\gamma|x_2|^{\gamma-1}$ ,  $\lambda_1 = 2(\eta - \delta)\beta\gamma|x_2|^{\gamma-1}$ . The following proof is the same as that of Theorem 1, so it will not be repeated.  $\square$

### 3.2 Guidance law design considering the dynamic delay of missile autopilot

Let  $a_m = x_3$ , we can get (26) from (4) and (14).

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{2\dot{r}}{r} & -\frac{1}{r} \\ 0 & 0 & -\frac{1}{\tau} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \\ &\begin{bmatrix} 0 \\ 0 \\ \frac{1}{\tau} \end{bmatrix} u + \begin{bmatrix} 0 \\ \frac{1}{r} \\ 0 \end{bmatrix} a_t. \end{aligned} \tag{26}$$

We select the sliding mode manifold

$$s_1 = k_0x_1 + x_2. \tag{27}$$

In order to reach the sliding mode manifold in finite time, as well as to reduce the chattering phenomenon of the control signal, we design the nonsingular terminal sliding mode manifold with linear sliding mode  $s_1$  and its derivative  $\dot{s}_1$  as

$$s_2 = s_1 + \beta_1\text{sig}^{\gamma_1}(\dot{s}_1) \tag{28}$$

where  $\dot{s}_1 = k_0\dot{x}_1 + \dot{x}_2 = k_0x_2 + \dot{x}_2, \ddot{s}_1 = k_0\dot{x}_2 + \ddot{x}_2$ .

Taking the derivative of the system of (28), we can get

$$\begin{aligned} \dot{s}_2 &= \dot{s}_1 + \beta_1\gamma_1|\dot{s}_1|^{\gamma_1-1}\ddot{s}_1 = \\ &\dot{s}_1 + \beta_1\gamma_1|\dot{s}_1|^{\gamma_1-1}\left(k_0\left(-\frac{2\dot{r}}{r}x_2 - \frac{1}{r}x_3 + \frac{1}{r}a_t\right) + \ddot{x}_2\right) \end{aligned} \tag{29}$$

$$\ddot{x}_2 = \frac{-2(\ddot{r}x_2 + \dot{r}\dot{x}_2)r + 2\dot{r}^2x_2}{r^2} + \frac{\dot{r}x_3 - \dot{x}_3r}{r^2} + \frac{r\dot{a}_t - \dot{r}a_t}{r^2}. \tag{30}$$

When (30) is substituted into (29)

$$\begin{aligned} \dot{s}_2 = \dot{s}_1 + \beta_1\gamma_1|\dot{s}_1|^{\gamma_1-1} & \left( \frac{-2r(k_0\dot{r} + \ddot{r}) + 6\dot{r}^2}{r^2}x_2 + \right. \\ & \left. \frac{3\dot{r} - k_0r}{r^2}x_3 + \frac{1}{r\tau}x_3 + \frac{(k_0a_t + \dot{a}_t)r - 3\dot{r}a_t}{r^2} - \frac{1}{r\tau}u \right). \end{aligned} \tag{31}$$

Let  $g_1(t) = \frac{(k_0a_t + \dot{a}_t)r - 3\dot{r}a_t}{r^2}$ , then

$$\begin{aligned} |g_1(t)| &= \left| \frac{(k_0a_t + \dot{a}_t)r - 3\dot{r}a_t}{r^2} \right| \leq \\ & \frac{k_0|a_t| + |\dot{a}_t|}{r_0} + \frac{3|\dot{r}a_t|}{r_0^2} = \\ & \frac{3A_1|V_t \cos(q - \phi_t) - V_m \cos(q - \phi_m)|}{r_0} + \frac{k_0A_1 + A_2}{r_0} \leq \\ & \frac{k_0A_1 + A_2}{r_0} + \frac{3A_1(|V_t^{\max}| + |V_m^{\max}|)}{r_0^2} \leq \varepsilon \end{aligned}$$

where  $V_t^{\max}$  and  $V_m^{\max}$  are the target's maximum speed and the missile's maximum speed respectively.

When the target is not maneuvering,  $a_t = 0$ . For the system of (26), we select rapid power reaching law

$$\dot{s}_2 = -k_5s_2 - k_6\text{sig}^{\rho_1}(s_2) \tag{32}$$

where,  $k_5, k_6 > 0, 0 < \rho_1 < 1$ .

We can get from (31) and (32) that

$$\begin{aligned} u = r\tau\beta_1^{-1}\gamma_1^{-1}|\dot{s}_1|^{2-\gamma_1} \text{sgn}(\dot{s}_1) - \\ 2k_0\tau\dot{r}x_2 + x_3 - 2\tau\ddot{r}x_2 - k_0\tau x_3 + \\ \frac{3\tau\dot{r}}{r}x_3 + r\tau\beta_1^{-1}\gamma_1^{-1}|\dot{s}_1|^{1-\gamma_1} (k_5s_2 + k_6\text{sig}^{\rho_1}(s_2)). \end{aligned} \tag{33}$$

For the factor  $\beta_1^{-1}\gamma_1^{-1}|\dot{s}_1|^{1-\gamma_1}$ , when  $\dot{s}_1 \rightarrow 0$ , it will result in singularity, thus we design the guidance law for the system of (26) as

$$\begin{aligned} u_1 = r\tau\beta_1^{-1}\gamma_1^{-1}|\dot{s}_1|^{2-\gamma_1} \text{sgn}(\dot{s}_1) - 2k_0\tau\dot{r}x_2 + x_3 - \\ 2\tau\ddot{r}x_2 - k_0\tau x_3 + \frac{3\tau\dot{r}}{r}x_3 + r\tau(k_5s_2 + k_6\text{sig}^{\rho_1}(s_2)). \end{aligned} \tag{34}$$

**Theorem 3.** For the system of (26), when  $a_t = 0$ , we can design the guidance law of (34) by selecting the sliding mode manifolds of (27)–(28) and considering the reaching law of (32), which allows the system to reach the nonsingular terminal sliding mode manifold  $s_2 = 0$  in finite time. Therefore, for the sliding mode manifold  $s_1 = \dot{s}_1 = 0$ , we can get that the system state  $x_1, x_2$  exponentially converges to zero, which means that the LOS angle exponentially converges to the desired LOS angle, and the LOS angle rate exponentially converges to zero.

**Proof.** When the guidance law of (34) is substituted into (31), where  $a_t = \dot{a}_t = 0$ , we can get

$$\dot{s}_2 = \beta_1\gamma_1|\dot{s}_1|^{\gamma_1-1}(-k_5s_2 - k_6\text{sig}^{\rho_1}(s_2)). \tag{35}$$

Construct Lyapunov function

$$V_2 = s_2^2. \tag{36}$$

Taking the derivative of (36), and substituting (35), we can get

$$\begin{aligned} \dot{V}_2 = 2s_2\dot{s}_2 = \\ 2\beta_1\gamma_1|\dot{s}_1|^{\gamma_1-1}s_2(-k_5s_2 - k_6\text{sig}^{\rho_1}(s_2)) = \\ 2\beta_1\gamma_1|\dot{s}_1|^{\gamma_1-1}(-k_5s_2^2 - k_6|s_2|^{\rho_1+1}) = \\ -\mu_2V_2 - \lambda_2V_2^{\frac{1+\rho_1}{2}} \leq 0 \end{aligned}$$

where  $\mu_2 = 2k_5\beta_1\gamma_1|\dot{s}_1|^{\gamma_1-1}$ ,  $\lambda_2 = 2k_6\beta_1\gamma_1|\dot{s}_1|^{\gamma_1-1}$ , and when  $\dot{s}_1 \neq 0$ , we can get from Lemma 1 that the system of (26) reaches the sliding mode manifold  $s_2 = 0$  in finite time. When,  $\dot{s}_1 = 0, s_2 \neq 0$ , we can get  $s_1 \neq 0$  from (28), but  $(s_1 \neq 0, \dot{s}_1 = 0)$  is not a stable equilibrium point, which means  $\dot{V}_2 = 0$  cannot be maintained; according to the reaching condition of sliding mode, the system will reach the stable equilibrium point, and retain the nonsingular terminal sliding mode state  $s_2 = 0$ . On the nonsingular terminal sliding mode manifold  $s_2 = 0$ , the system state  $s_1, \dot{s}_1$  converges to zero in finite time. As  $s_1 = k_0x_1 + x_2 = 0, \dot{s}_1 = k_0\dot{x}_1 + \dot{x}_2 = k_0x_2 + \dot{x}_2 = 0$ , we can get that  $x_1, x_2$  exponentially converges to zero, which means that the LOS angle exponentially converges to the desired value, and the LOS angle rate exponentially converges to zero.  $\square$

When the target is maneuvering, we select an exponential reaching law for the sliding mode manifold of (28):

$$\dot{s}_2 = -k_7s_2 - k_8\text{sgn}(s_2) \tag{37}$$

where  $k_7, k_8 > 0$ .

For the system of (26), we can design the guidance law as

$$\begin{aligned} u_2 = r\tau\beta_1^{-1}\gamma_1^{-1}|\dot{s}_1|^{2-\gamma_1} \text{sgn}(\dot{s}_1) - 2k_0\tau\dot{r}x_2 + x_3 - \\ 2\tau\ddot{r}x_2 - k_0\tau x_3 + \frac{3\tau\dot{r}}{r}x_3 + r\tau(k_7s_2 + \zeta\text{sgn}(s_2)) \end{aligned} \tag{38}$$

where  $\zeta \geq k_8 + \varepsilon$ .

**Theorem 4.** For the guidance law of (38), when  $\zeta \geq k_8 + \varepsilon, \varepsilon = \text{const.} > 0$ , this guidance law can compensate the target's maneuvering in the guidance system of (26), and make the system reach the sliding mode manifold  $s_2 = 0$  in finite time. Therefore, for the sliding mode manifold  $s_1 = \dot{s}_1 = 0$ , we can get that the system state  $x_1, x_2$  exponentially converges to zero, which means that the LOS angle exponentially converges to the desired LOS angle, and the LOS angle rate exponentially converges to zero.

**Proof.** When the guidance law of (38) is substituted into (31), we can get

$$\dot{s}_2 = \beta_1\gamma_1|\dot{s}_1|^{\gamma_1-1}(-k_7s_2 - \zeta\text{sgn}(s_2) + g_1(t)). \tag{39}$$

Construct Lyapunov function

$$V_3 = s_2^2. \tag{40}$$

Taking the derivative of (40), and substituting (39), we can get

$$\begin{aligned} \dot{V}_3 &= 2s_2\dot{s}_2 = \\ &\beta_1\gamma_1|\dot{s}_1|^{\gamma_1-1}(-k_7s_2 - \zeta\text{sgn}(s_2) + g_1(t)) = \\ &2\beta_1\gamma_1|\dot{s}_1|^{\gamma_1-1}(-k_7s_2^2 - \zeta|s_2| + g_1(t)s_2) \leq \\ &2\beta_1\gamma_1|\dot{s}_1|^{\gamma_1-1}(-k_7s_2^2 - \zeta|s_2| + |g_1(t)s_2|) \leq \\ &2\beta_1\gamma_1|\dot{s}_1|^{\gamma_1-1}(-k_7s_2^2 - (\zeta - \varepsilon)|s_2|) = \\ &-\mu_3V_1 - \lambda_3V_1^{\frac{1}{2}} \leq 0 \end{aligned} \tag{41}$$

where  $\mu_3 = 2k_7\beta_1\gamma_1|\dot{s}_1|^{\gamma_1-1}$ ,  $\lambda_3 = 2(\zeta - \varepsilon)\beta_1\gamma_1|\dot{s}_1|^{\gamma_1-1}$ . The following proof is the same as that of Theorem 1, so it will not be repeated.  $\square$

**Remark 2.** In practical applications, the rate of relative range  $\dot{r}$  can be approximately viewed as a constant. Therefore, we have  $\ddot{r} \approx 0$ . Then, the proposed guidance law (34) can be rewritten as (42).

$$\begin{aligned} u_1 &= r\tau\beta_1^{-1}\gamma_1^{-1}|\dot{s}_1|^{2-\gamma_1}\text{sgn}(\dot{s}_1) - 2k_0\tau\dot{r}x_2 + x_3 - \\ &k_0\tau x_3 + \frac{3\tau\dot{r}}{r}x_3 + r\tau(k_5s_2 + k_6\text{sig}^{\rho_1}(s_2)). \end{aligned} \tag{42}$$

And the proposed guidance law (38) can be rewritten as (43).

$$\begin{aligned} u_2 &= r\tau\beta_1^{-1}\gamma_1^{-1}|\dot{s}_1|^{2-\gamma_1}\text{sgn}(\dot{s}_1) - 2k_0\tau\dot{r}x_2 + x_3 - \\ &k_0\tau x_3 + \frac{3\tau\dot{r}}{r}x_3 + r\tau(k_7s_2 + \zeta\text{sgn}(s_2)). \end{aligned} \tag{43}$$

**Remark 3.** When the target is not maneuvering, we select rapid power reaching law, which can not only improve the speed to reach the sliding mode manifold, but also inhibit its chattering behavior.

**Remark 4.** When the target is maneuvering, we select exponential reaching law, and adjust the term of variable structure, to compensate the disturbance to the target in the system. The convergence rate of exponential reaching law is quite fast, while there will be greater system chattering when the system state is near sliding mode. High-order sliding mode can alleviate the chattering resulted from sign function; however, in order to eliminate chattering, we further replace the sign function in the designed guidance law with the saturation function  $\text{sat}(s)$ , and the resulted form is as follows:

$$\text{sat}(s) = \begin{cases} 1, & s > \Delta \\ \frac{s}{\Delta}, & |s| \leq \Delta \\ -1, & s \leq -\Delta \end{cases}$$

where  $\Delta$  is the boundary layer.

### 4 Numerical simulation

Taking a missile air intercept as an example, in the reference inertial coordinate system, the velocities of the missile and target are constants, i.e., 600 m/s and 200 m/s, respectively. The missile's initial position is  $x_m(0) = -3$  km,  $y_m(0) = 10$  km, and its initial flight path angle is  $\phi_m(0) = -30^\circ$ ; the target's initial position is  $x_t(0) = 0$  km,

$y_t(0) = 0$  km, and its initial flight path angle  $\phi_t(0) = 135^\circ$ . The guidance distance of the seeker is  $r_0 = 100$  m. The desired LOS angle is  $q_d = -80^\circ$ , and the missile acceleration is limited not to exceed  $40g$ ,  $g=9.8$  m/s<sup>2</sup>. Figs. 2(a)–2(d) show the LOS angular rate, LOS angle, sliding mode surface, and missile normal acceleration, respectively.

#### 4.1 Not considering the dynamic delay of missile autopilot

We verify the effectiveness of the proposed guidance law (20) and guidance law (24), without considering the dynamic delay of missile autopilot. During simulation, the parameters of the guidance law are selected as:  $\beta = 5$ ,  $\gamma = \frac{5}{3}$ ,  $k_1 = 0.6$ ,  $k_2 = 0.25$ ,  $\rho = \frac{2}{3}$ ,  $k_3 = 2$ ,  $\Delta = 0.01$ ,  $\eta = 65$ .

Here, under the proposed guidance law (20), we consider that the missile intercepts a non-maneuvering target. The responses of LOS angular rate, LOS angle, sliding mode surface and missile normal acceleration are shown in Figs. 2(a)–2(d), respectively. From Figs. 2(a) and 2(b), it can be seen that the LOS angular rate converges to zero fast in finite time and the LOS angle converges to the desired LOS angle in finite time. So, the proposed guidance law can guarantee the missile intercepts the target with the desired LOS angle successfully. From Fig. 2(c), we can observe that the sliding mode surface without chattering reaches to zero in finite time. As shown in Fig. 2(d), the proposed guidance law can guarantee the missile normal acceleration converges to zero fast, and there is no chattering phenomenon. Under the proposed guidance law, the interception time, miss-distance and LOS angle error are 15.018 s, 0.328 m and  $0.235^\circ$ , respectively, which are also given to illustrate the effectiveness of the proposed guidance law (20).

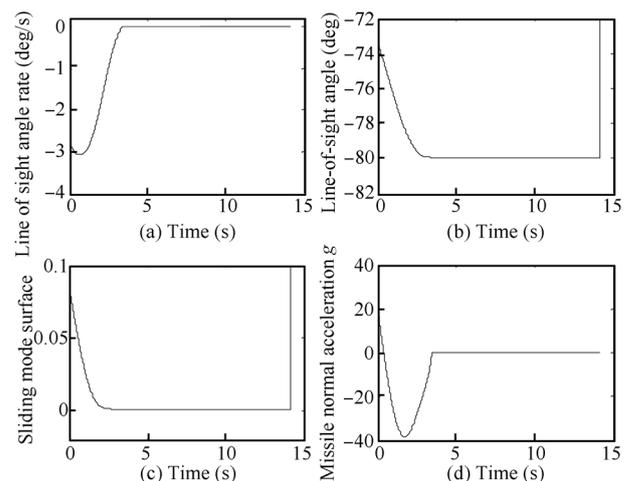


Fig.2 Responses under the guidance law (20) for non-maneuvering target

For the case without considering the dynamic delay of missile autopilot, the simulation results are shown in Figs. 2(a)–2(d). In practical problems, the dynamic delay of missile autopilot is not ignorable. Hence, we select  $\tau = 0.5$

to demonstrate the effectiveness of the guidance law (20) for the case when the dynamic delay of missile autopilot exists. From Figs. 3(a)–3(d), it can be seen that the guidance law (20) cannot satisfy the requirements of guidance performance.

To illustrate the performance of the designed guidance law (24), we consider three different target acceleration profiles as given below.

Case 1. Cosine maneuvering  $5g \cos\left(\pi \frac{t}{4}\right)$ .

Case 2. Step maneuvering  $5g$ .

Case 3. Constant maneuvering  $5g$ .

The miss-distances, LOS angle errors and interception times are given in Table 1. The curves of LOS angular rate, LOS angle, sliding mode surface and missile normal acceleration are shown in Figs. 4(a)–4(d), respectively.

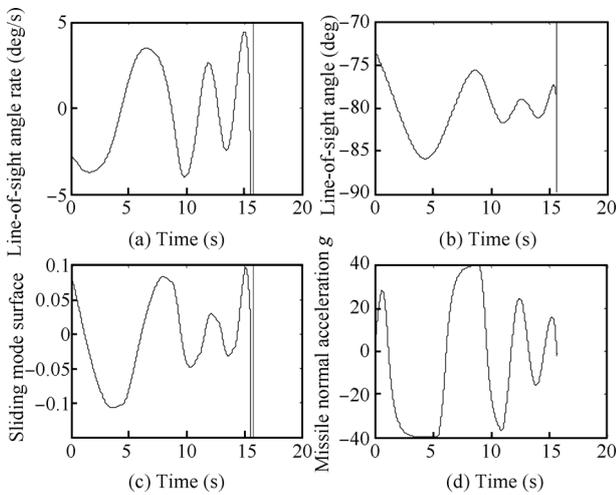


Fig. 3 Responses under the guidance law (20) for considering the dynamic delay of missile autopilot

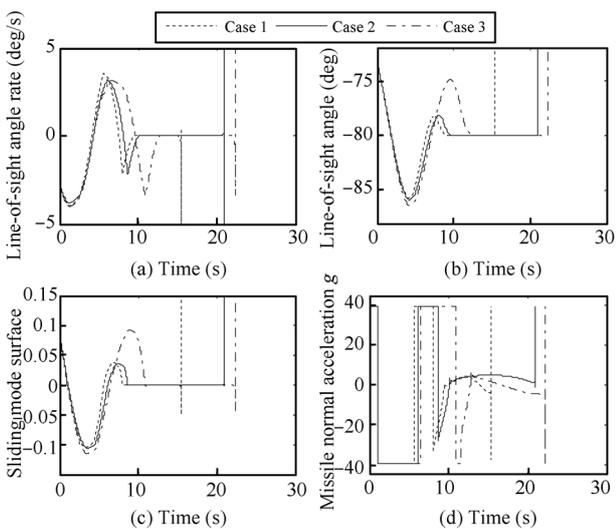


Fig. 4 Responses under the guidance law (24) for three cases

Table 1 Corresponding data for the three target acceleration profiles

	Miss-distances (m)	LOS angle errors (°)	Interception times (s)
Case 1	0.055	0.063	15.442
Case 2	0.169	0.040	20.972
Case 3	0.317	0.074	22.321

From Figs. 4(a)–4(c), in the three target acceleration profiles, we can see that the LOS angular rates and sliding mode surfaces converge to zero fast. But, for the three cases, the curves of the LOS angular rates have peak, which lead to small peak in the curves of the missile normal accelerations shown in Fig. 4 (d). From Fig. 4 (d), we can observe that there are acceleration saturations before 10 s in all the three cases, while the accelerations are decreasing correspondingly as the LOS angular rates tend to zero. From Fig. 4 (b), the proposed guidance law can guarantee the LOS angles converge to the desired LOS angle. From Table 1, simulation data verify the high precision guidance performance of the proposed guidance law (24).

For the case without considering the dynamic delay of missile autopilot, the simulation results are shown in Fig. 4 to demonstrate the effectiveness of the guidance law (24). Further, for the case 1, the parameter  $\tau$  is chosen as  $\tau = 0.5$ . From Figs. 5(a)–5(d), it can be seen that the guidance law (24) cannot satisfy the requirements of guidance while considering the dynamic delay of missile autopilot.

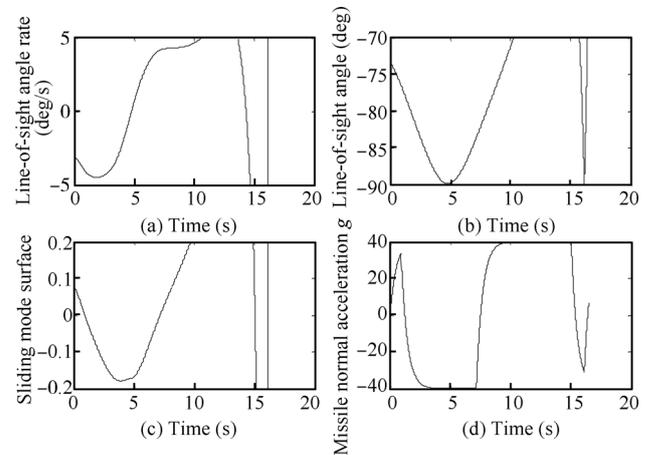


Fig. 5 Responses under the guidance law (24) for considering the dynamic delay of missile autopilot

### 4.2 Considering the dynamic delay of missile autopilot

We verify the effectiveness of the designed guidance laws (42) and (43), considering the dynamic delay of missile autopilot. During simulation, the parameters of the guidance law are selected as:  $\beta_1 = 5$ ,  $\gamma_1 = \frac{5}{3}$ ,  $k_0 = 3$ ,  $k_5 = 0.5$ ,  $k_6 = 0.2$ ,  $\rho_1 = \frac{2}{3}$ ,  $\zeta = 65$ ,  $k_7 = 3$ ,  $\Delta = 0.01$ ,  $\tau = 0.5$ .

For a non-maneuvering target, the proposed guidance law (42) is applied. The responses of LOS angular rate, LOS

angle, sliding mode surface and missile normal acceleration are shown in Figs. 6(a)–6(d), respectively. From Figs. 6(a)–6(c), it can be seen that the LOS angular rate and sliding mode surface converge to zero fast. As shown in Fig. 6(b), the proposed guidance law can guarantee the LOS angle converges to the desired LOS angle and the LOS angle error is zero. As shown in Fig. 6(d), while considering the dynamic delay of missile autopilot, the proposed guidance law can guarantee the missile normal acceleration converges to zero fast and there is no chattering phenomenon. Under the proposed guidance law (42), the interception time, miss-distance and LOS angle error are 21.68 s, 0.02 m and 0.05°. It can be obtained that the guidance precision of the guidance law (42) is higher than that of guidance law (20).

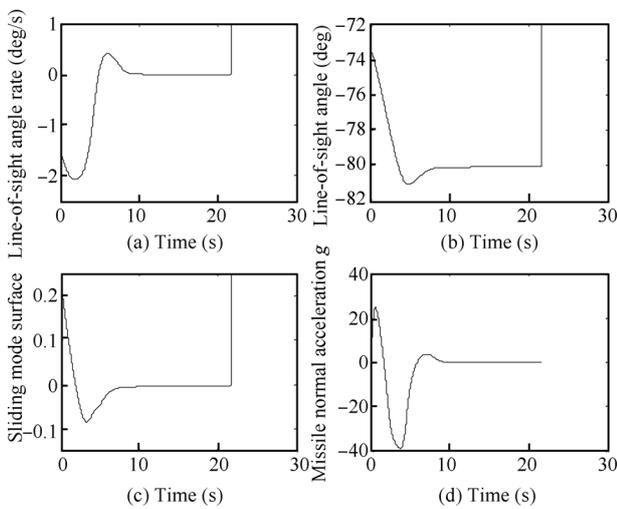


Fig. 6 Responses under the guidance law (42) for non-maneuvering target

To verify the performance of the designed guidance law (43) for the maneuvering target, we also consider the above-mentioned three different target acceleration profiles. The

miss-distances, LOS angle errors and interception times are given in Table 2. The curves of LOS angular rate, LOS angle, sliding mode surface and missile normal acceleration are shown in Figs. 7(a)–7(d), respectively.

Table 2 Corresponding data for three target acceleration profiles

	Miss-distances (m)	LOS angle errors (°)	Interception times (s)
Case 1	0.042	0.01	14.408
Case 2	0.083	0.02	18.908
Case 3	0.020	0.03	20.241

From the Figs. 7(a) and 7(c), in the three target acceleration profiles, we can see that the LOS angular rates and sliding mode surfaces converge to zero fast. In addition, we can also observe that the curves of LOS angular rates considering the missile’s dynamic delay are smoother than those without considering dynamic characteristics. As shown in Fig. 7(d), the saturation times of corresponding missile normal accelerations are shorter, then these accelerations decrease rapidly and there are no chattering phenomenon in the three cases. From Fig. 7(b), the proposed guidance law can guarantee the LOS angles converge to the desired LOS angle for three cases. From the Table 2, simulation data verify the high precision guidance performance of the proposed guidance law (43).

### 4.3 Comparing with the other guidance laws

At present, much study has been devoted to the guidance law design about the dynamic delay and impact angle constraints. However, there are not many guidance laws which are designed by applying the non-singular terminal sliding mode control. In [20], a guidance law based on the non-linear backstepping method with autopilot lag and impact angle constraint was proposed, which was expressed as

$$A_c = \frac{\tau}{\cos \theta_m} \left\{ \begin{aligned} & \left( \frac{k_1 \cos^2 \theta_m}{R(t)} + c_1 k_1 k_2 |\dot{R}(t)| \right) x_1 + \left( \frac{1}{\tau} - \frac{(k_2 + 2) |\dot{R}(t)|}{R(t)} - k_1 - c_1 \right) x_3 \cos \theta_m + \\ & \left( \frac{\cos^2 \theta_m}{R(t)} + c_1 (2 + k_2) |\dot{R}(t)| + c_1 k_1 R(t) + k_1 (1 + k_2) |\dot{R}(t)| + \frac{(2k_2 + 4) |\dot{R}(t)|^2}{R(t)} \right) x_2 + \\ & c_1 \varepsilon_1 \text{sat}(z_3) - \left( \frac{(k_2 + 2) |\dot{R}(t)|}{R(t)} + k_1 \right) \varepsilon_1 \text{sgnz}_4 \end{aligned} \right\} \quad (44)$$

where  $z_3 = x_2 + k_1 x_1, k_1 = 3, k_2 = 2, c_1 = 30, \varepsilon_1 = 40$ .

For Case 1, the initial conditions are chosen as the same as in the previous simulation. Simulation results are shown in Figs. 8(a)–8(d) for the guidance laws (44) and (43).

From Figs. 8(a) and 8(b), the LOS angular rate and LOS angle under the guidance law (43) faster converge to their corresponding desired values than that under the guidance

law (44). In addition, the convergence accuracy under the guidance law (43) is higher than that under the guidance law (44). As shown in Fig. 8(c), it can be obtained that the convergence rate of the guidance law (43) is faster than that of the guidance law (44). From Fig. 8(d), although the missile normal accelerations for the two guidance laws are similar, the chattering problem of the missile normal accel-

eration under the guidance law (44) is a little bit serious.

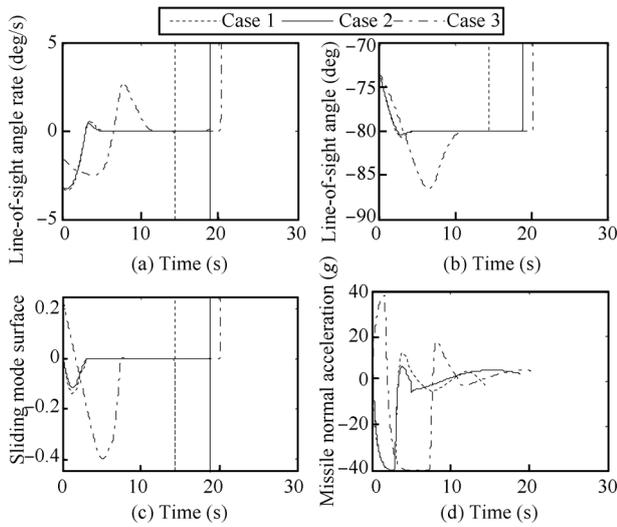


Fig. 7 Responses under the guidance law (43) for three cases

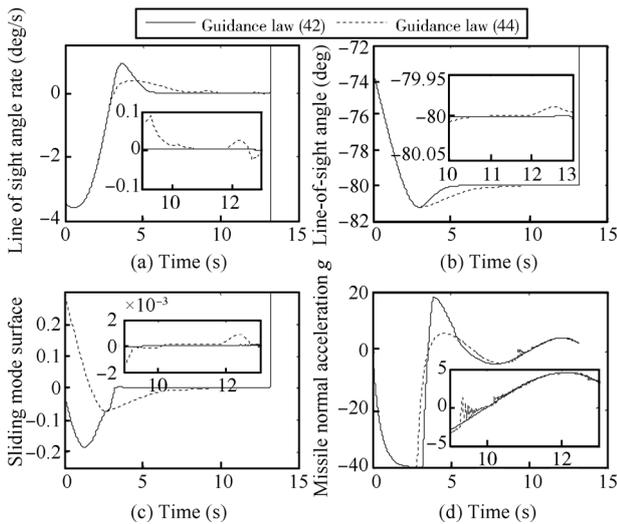


Fig. 8 Responses under the guidance law (43) and guidance law (44) for Case 1

## 5 Conclusions

While not considering the missile's dynamic delay, we combine the nonsingular terminal sliding mode and corresponding reaching law to design a finite-time convergent guidance law which meets both the impact angle constraints and zero miss distance. While considering the missile's dynamic delay, we combine the high-order nonsingular terminal sliding mode and corresponding reaching law to design an exponential convergent guidance law which meets both the impact angle constraints and zero miss distance. The numerical simulation is performed for different maneuvering maneuvers of the target, which verifies the effectiveness of the guidance law designed in this paper.

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